# GP1-L04

**Problem 1**

An ion's position vector is initially $\vec{r} = 5\hat{i} - 6\hat{j} + 2\hat{k}$ , and 10 s later is $\vec{r} = -2\hat{i} + 8\hat{j} - 2\hat{k}$ , all in meters. In unit-vector notation, what is its average velocity $\vec{v}_{ave}$ during the 10 s? $\vec{v}_{ave} = a\hat{i} + b\hat{j} + c\hat{k}$ (m/s$^2$)
(Hint:Use the definition of average velocity in vector form.)

$$\vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t}$$

$$v_{avg} = \frac{(-2\hat{i} + 8\hat{j} - 2\hat{k}m) - (5\hat{i} - 6\hat{j} + 2\hat{k})m}{10s}$$

$$= \frac{(-7\hat{i} + 14\hat{j} - 4\hat{k})m}{10s}$$

$$= (-0.7\hat{i} + 1.4\hat{j} - 0.4\hat{k})m/s$$

**Problem 2**

A cart is propelled over an xy plane with acceleration components $a_x = 4.0\text{m/s}^2$ and $a_y = -2.0\text{m/s}^2$. Its initial velocity has components $v_{0x} = 8.0$ m/s and $v_{0y} = 12$ m/s. In unit-vector notation, what is the velocity of the cart when it reaches its greatest y coordinate? $\vec{v} = v_x\hat{i} + v_y\hat{j}$ m/s.

(a) $v_x$=_____ m/s

04: **ANS:=32**

(b) $v_y$=_____ m/s

05: **ANS:=0**

$\vec{a} = (4, -2) =$ const vector

$\vec{v} = \vec{v}_0 + \vec{a}t$ , $\vec{v}_0 = (8, 12)$

$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$

$\vec{v}$: $\begin{cases} x: v_x = v_{0x} + a_x t = 8 + 4t \\ y: v_y = v_{0y} + a_y t = 12 - 2t \end{cases}$

$\vec{r}$: $\begin{cases} x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \\ y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 \end{cases}$ Let's take $\vec{r}_0 = (0, 0)$

$y = 12t + \frac{1}{2}(-2)t^2$ 極值 $\Rightarrow \frac{dy}{dt} = 0$

$\frac{dy}{dt} = 12 - 2t = 0 \Rightarrow t = 6$

$v_x = 8 + 4t = 8 + 24 = 32$

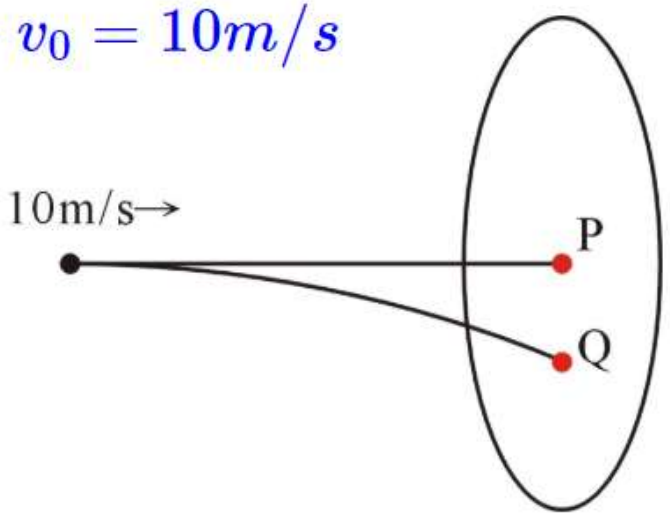
$v_y = 12 - 2t = 12 - 12 = 0$

**Problem 3**

A dart is thrown horizontally with an initial speed of 10 m/s toward point P, the bull's-eye on a dart board. It hits at point Q on the rim, vertically below P, 0.19 s later. (a) what is the distance $\bar{PQ}$? (b) How far away from the dart board is the dart released?

$$v_{0y} = 0$$

$$v_{0x} = v_0 = 10m/s$$




$$(a)\ y = -\frac{1}{2}gt^2$$

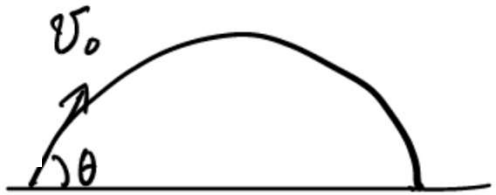
$$y = -PQ$$

$$PQ = \frac{1}{2}(9.8m/s^2)(0.19s)^2 = 0.18m$$

$$(b)\ x = v_0 t = (10m/s)(0.19s) = 1.9m$$

**Problem 4**

If a baseball player can throw a ball at $45^0$ to a point 100 m away horizontally to the initial vertical level, how high could he throw it vertically upward?




抛体运动的基本公式：

初始：$v_{0x} = v_0 \cos\theta$

$v_{0y} = v_0 \sin\theta$

铅垂：$v_y = v_{0y} - gt$

飞至最高点的时间 $t_H$

$0 = v_{0y} - g t_H$

$t_H = \frac{v_0 \sin\theta}{g}$

落地的时间 $t_f$

$t_f = 2t_H = \frac{2v_0 \sin\theta}{g}$

最高点的高度 H：

$H = v_{0y} t_H - \frac{1}{2} g t_H^2$

$= v_0 \sin\theta \cdot \frac{v_0 \sin\theta}{g} - \frac{1}{2} g \left(\frac{v_0 \sin\theta}{g}\right)^2$

$= \frac{v_0^2 \sin^2\theta}{g} - \frac{v_0^2 \sin^2\theta}{2g}$

$= \frac{(v_0 \sin\theta)^2}{2g}$

落地的水平距离 = 射程

$R = v_x t_f$

$= v_0 \cos\theta \cdot \frac{2v_0 \sin\theta}{g}$

$= \frac{v_0^2 (2 \sin\theta \cos\theta)}{g}$

$= \frac{v_0^2 \sin 2\theta}{g}$

$$R = \frac{v_0^2 \times sin(2\theta)}{g}$$

$$v_0 = 31.3 m/s$$

$$H = \frac{v_0^2}{2g} = 50m$$

**Problem 5**

A certain airplane has a speed of $290$ km/h and diving at an angle of $\theta = 30^0$ below the horizontal when the pilot releases a radar decoy (The figure). The horizontal distance between the release point and the point where the decoy strikes the ground is $d = 700$ m. (a) How long is the decoy in the air? (b) How high was the release point?

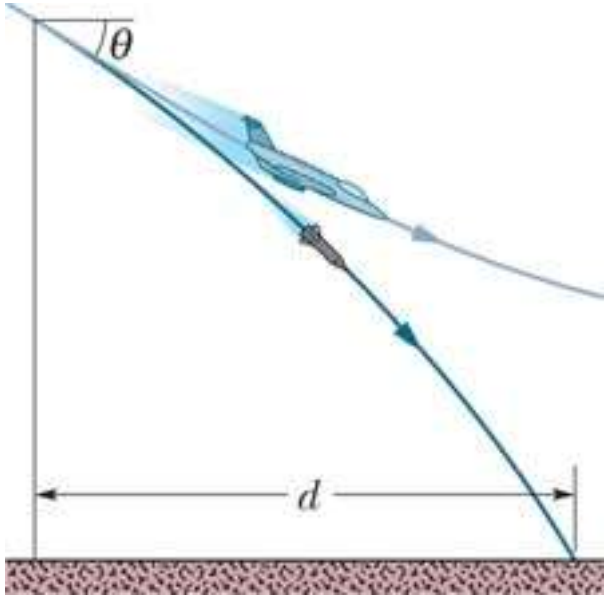
$$v_0 = 290km/h = 290km/h(\frac{1000m}{km})(\frac{1h}{3600s}) = 80.6m/s$$

$$\Delta x = (v_0 cos\theta_0)t \qquad t = \frac{700m}{(80.6m/s)cos - 30^0} = 10s$$

$$y - y_0 = (v_0 sin\theta_0)t - \tfrac{1}{2}gt^2$$

$$0 - y_0 = (-40.3m/s)(10s) - \tfrac{1}{2}(9.8m/s^2)(10s)^2$$

$$y_0 = 897m$$



**Problem 6**

A projectiles launch speed is five times its speed at maximum height. Find launch angle $\theta_0$.

最高點時$v_x = v_0 cos\theta_0 = v$

$v_y = 0$

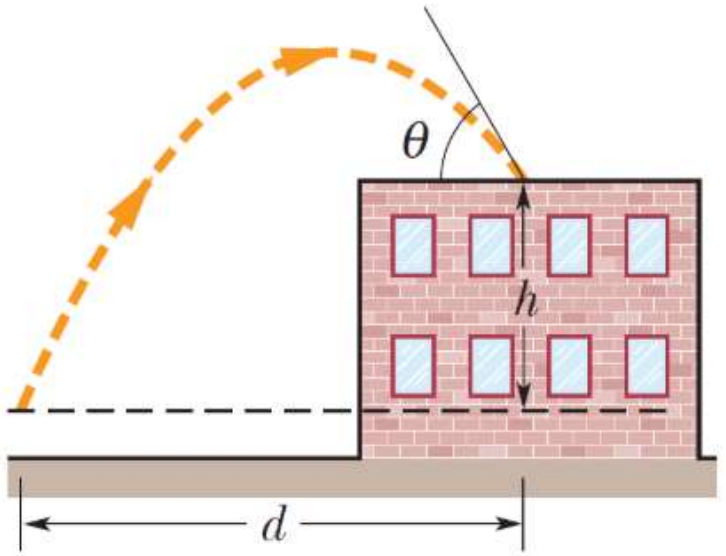
啟動速度$v_0 = 5v$

$(5v)cos\theta_0 = v$

$\theta_0 = cos^{-1}\frac{1}{5} = 78.6^0$

**Problem 7**

In the figure, a ball is thrown up onto a roof, landing 4.00 s later at height h = 20.0 m above the release level. The ball's path just before landing is angled at $\theta = 60^o$ with the roof. (a) Find the horizontal distance d it travels. What are the (b) magnitude and (c) angle (relative to the horizontal) of the ball's initial velocity?




斜向拋体運动

從屋頂射出落至地面

射角 = 60°

$v_{oy} \cdot 4 - \frac{1}{2} g (4)^2 = -20$

$\Rightarrow v_{oy} = 14.6 = v_0 \sin 60°$

$\Rightarrow v_0 = 16.86$

$v_y = v_{oy} - g(4)$

$= 14.6 - 9.8 \cdot 4$

$= -24.6$

$v_x = v_0 \cos\theta = 8.43$

$d = v_x \cdot t = 8.43 \cdot 4$

$= 33.72$

$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{24.6^2 + 8.43^2}$

$= 26.00$

$\theta = \tan^{-1} \frac{24.6}{8.43} = 71.08°$

**Problem 8**

particle moves horizontally in uniform circular motion, over a horizontal xy plane. At one instant, it moves through the point at coordinates (4.00 m, 4.00 m) with a velocity of $\vec{v} = -5.00\hat{i}$ m/s and an acceleration of $\vec{a} = 12.5\hat{j}$ m/s$^2$. What are the (a) x and (b) y coordinates of the center of the circular path and (c)the frequency of the circular motion? (03小題)

(a)x coordinate of the center=____ m

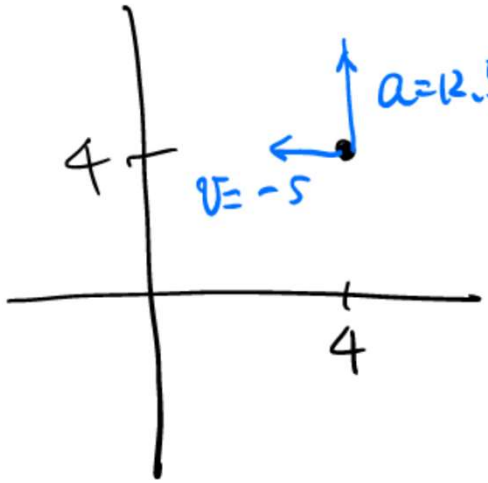
15: **ANS:=4**

(b)y coordinate of the center=____ m

16: **ANS:=6**

(c)frequency=____ Hz

17: **ANS:=0.398**




"向心力" ⇒ $X_c = 4$

圓心在 (4,4) 的正上方

⇒ 決定 r=?

$a = \frac{v^2}{r}$

$r = \frac{v^2}{a} = \frac{5^2}{12.5} = 2$

⇒ 圓心 $= (4, 4+2) = (4, 6)$

$T = \frac{2\pi r}{v} = \frac{2\pi \cdot 2}{5} = 2.513(s)$

$f_r = \frac{1}{T} = 0.398(Hz)$

**Problem 9**

當一顆大恒星變成超新星時，它的核心可能被壓縮得如此緊密，以至於它變成了一個半徑約20公里的中子星。如果一顆中子星每秒旋轉一周，（a）在中子星赤道上的粒子的旋轉速度是多少？（b）粒子的向心加速度的大小是多少？ (02小題)

(a)粒子的旋轉速度=_____ m/s

18: **ANS:=1.26E5**

（b）粒子的向心加速度=_____ $m/s^2$

19: **ANS:=7.9E5**

每秒1週 $\Rightarrow T=1$

$$v=\frac{2\pi r}{T}=\frac{2\pi\cdot 20\times 10^3}{1}=1.257\times 10^5\ m/s$$

$$a=\frac{v^2}{r}=\frac{(1.257\times 10^5)^2}{2\times 10^4}=7.90\times 10^5\ m/s^2$$

**Problem 10**

A rotating fan completes 1200 revolutions every minute. Consider the tip of a blade, at a radius of 0.15 m. (a) Through what distance does the tip move in one revolution? What are (b) the tip's speed and (c) the magnitude of its acceleration? (d) What is the period of the motion? (04小題)

(a)the distance=_____ m

20: **ANS:=0.94**

(b)tip's speed=_____ m/s

21: **ANS:=19**

(c)acceleration, a=_____ m/

22: **ANS:=2.4E3**

(d)period=_____ s

23: **ANS:=0.05**

半徑 $r=0.15$

轉1圈所運動的距離=圓周長$=2\pi r$

$=2\pi \cdot 0.15 = 0.942(m)$

周期：每分鐘轉1200圈 $\Rightarrow T=\frac{60}{1200}=0.05(s)$

$v=\frac{2\pi r}{T}=\frac{0.942}{0.05}=18.85(m/s)$

$a=\frac{v^2}{r}=\frac{18.85^2}{0.15}=2.369\times10^3$