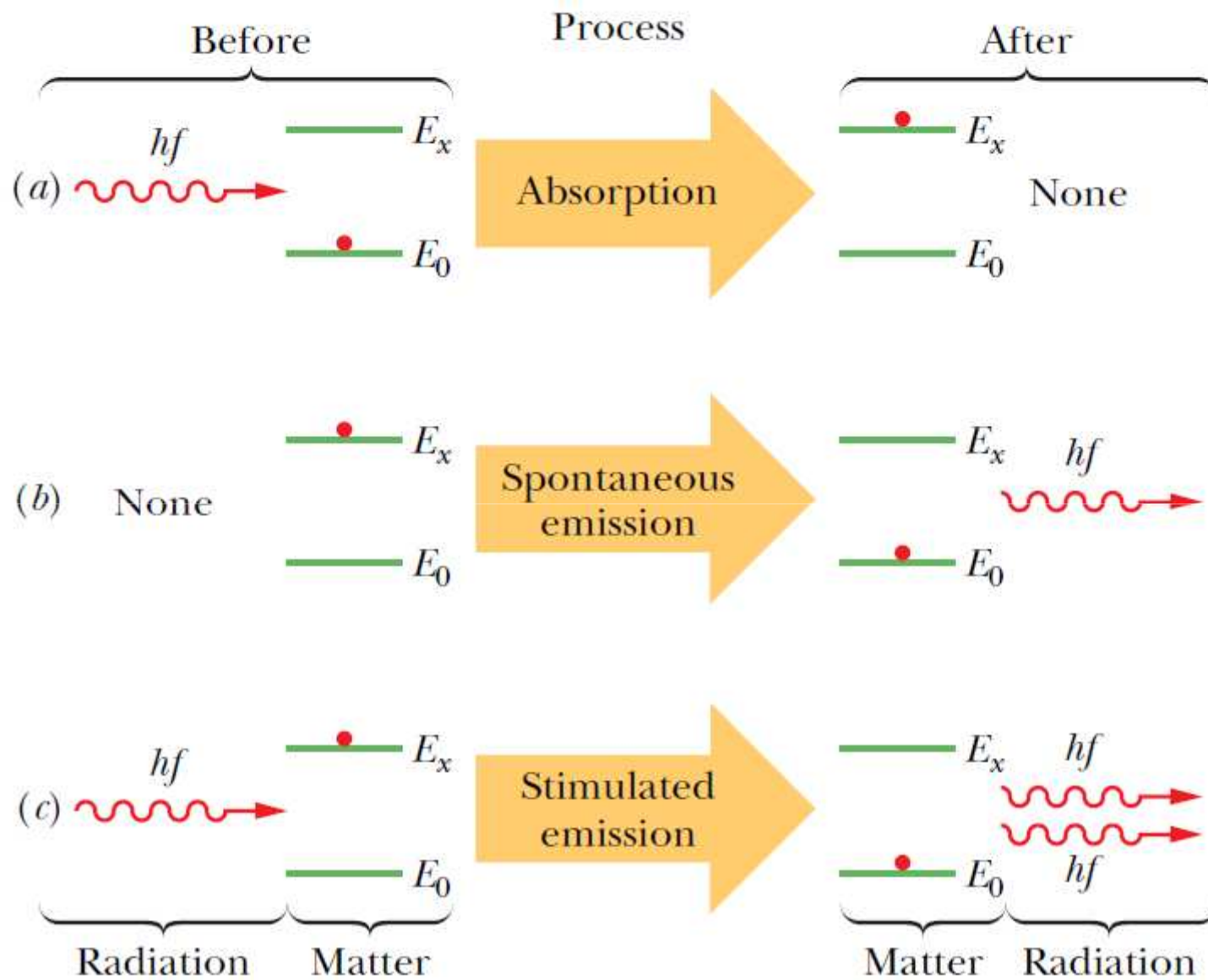


1. ***Laser light is highly monochromatic.*** Light from an ordinary incandescent lightbulb is spread over a continuous range of wavelengths and is certainly not monochromatic. The radiation from a fluorescent neon sign is monochromatic, true, to about 1 part in 10^6 , but the sharpness of definition of laser light can be many times greater, as much as 1 part in 10^{15} .
2. ***Laser light is highly coherent.*** Individual long waves (*wave trains*) for laser light can be several hundred kilometers long. When two separated beams that have traveled such distances over separate paths are recombined, they “remember” their common origin and are able to form a pattern of interference fringes. The corresponding *coherence length* for wave trains emitted by a lightbulb is typically less than a meter.
3. ***Laser light is highly directional.*** A laser beam spreads very little; it departs from strict parallelism only because of diffraction at the exit aperture of the laser. For example, a laser pulse used to measure the distance to the Moon generates a spot on the Moon’s surface with a diameter of only a few meters. Light from an ordinary bulb can be made into an approximately parallel beam by a lens, but the beam divergence is much greater than for laser light. Each point on a lightbulb’s filament forms its own separate beam, and the angular divergence of the overall composite beam is set by the size of the filament.
4. ***Laser light can be sharply focused.*** If two light beams transport the same amount of energy, the beam that can be focused to the smaller spot will have the greater intensity at that spot. For laser light, the focused spot can be so small that an intensity of 10^{17} W/cm^2 is readily obtained. An oxyacetylene flame, by contrast, has an intensity of only about 10^3 W/cm^2 .



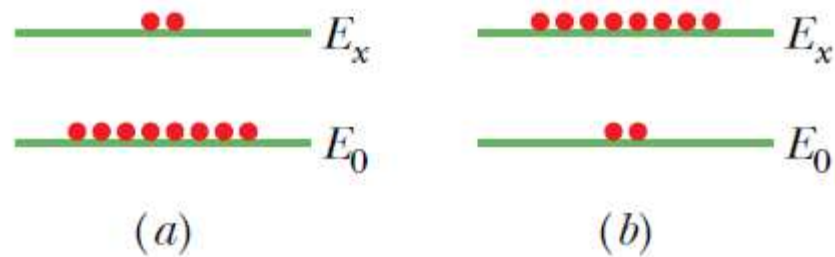
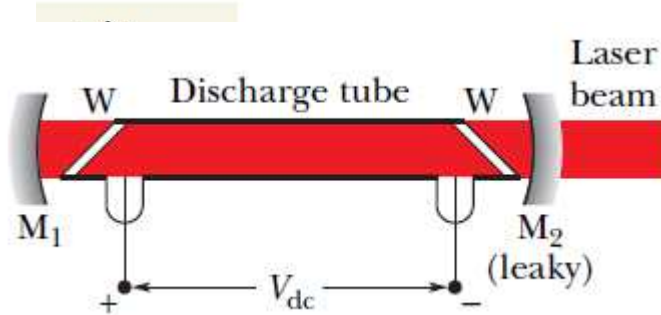


FIG. 40-20 (a) The equilibrium distribution of atoms between the ground state E_0 and excited state E_x accounted for by thermal agitation. (b) An inverted population, obtained by special methods. Such a population inversion is essential for laser action.

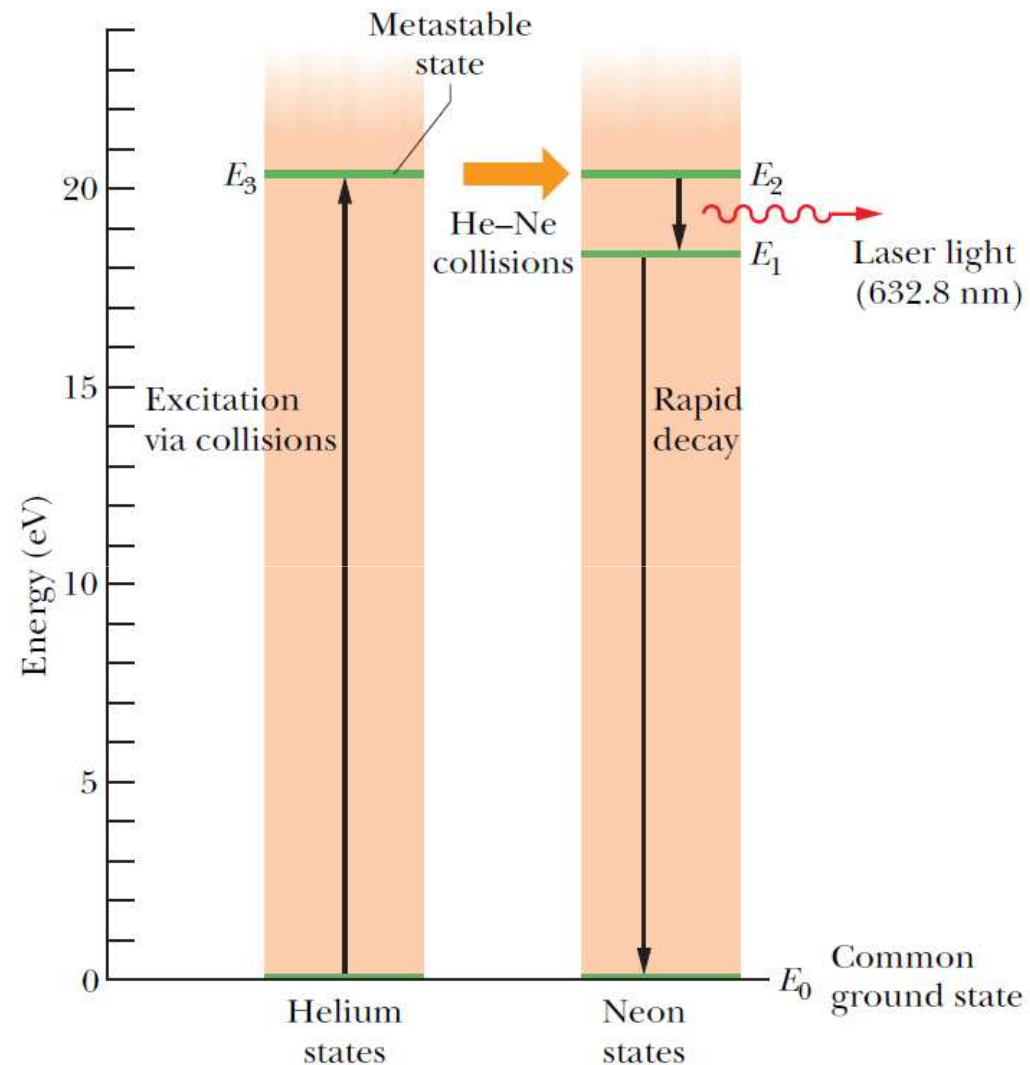
$$N_x = N_0 e^{-(E_x - E_0)/kT},$$

The Helium–Neon Gas Laser



The glass discharge tube is filled with a 20 : 80 mixture of helium and neon gases, neon being the medium in which laser action occurs.

FIG. 40-22 Five essential energy levels for helium and neon atoms in a helium–neon gas laser. Laser action occurs between levels E_2 and E_1 of neon when more atoms are at the E_2 level than at the E_1 level.



In the helium–neon laser of Fig. 40-21, laser action occurs between two excited states of the neon atom. However, in many lasers, laser action (*lasing*) occurs between the ground state and an excited state, as suggested in Fig. 40-20*b*.

(a) Consider such a laser that emits at wavelength $\lambda = 550$ nm. If a population inversion is not generated, what is the ratio of the population of atoms in state E_x to the population in the ground state E_0 , with the atoms at room temperature?

$$\begin{aligned} E_x - E_0 &= hf = \frac{hc}{\lambda} \\ &= \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(3.00 \times 10^8 \text{ m/s})}{(550 \times 10^{-9} \text{ m})(1.60 \times 10^{-19} \text{ J/eV})} \\ &= 2.26 \text{ eV}. \end{aligned}$$

$$kT = (8.62 \times 10^{-5} \text{ eV/K})(300 \text{ K}) = 0.0259 \text{ eV},$$

$$N_x/N_0 = e^{-(E_x - E_0)/kT} \qquad N_x/N_0 = e^{-(2.26 \text{ eV})/(0.0259 \text{ eV})}$$

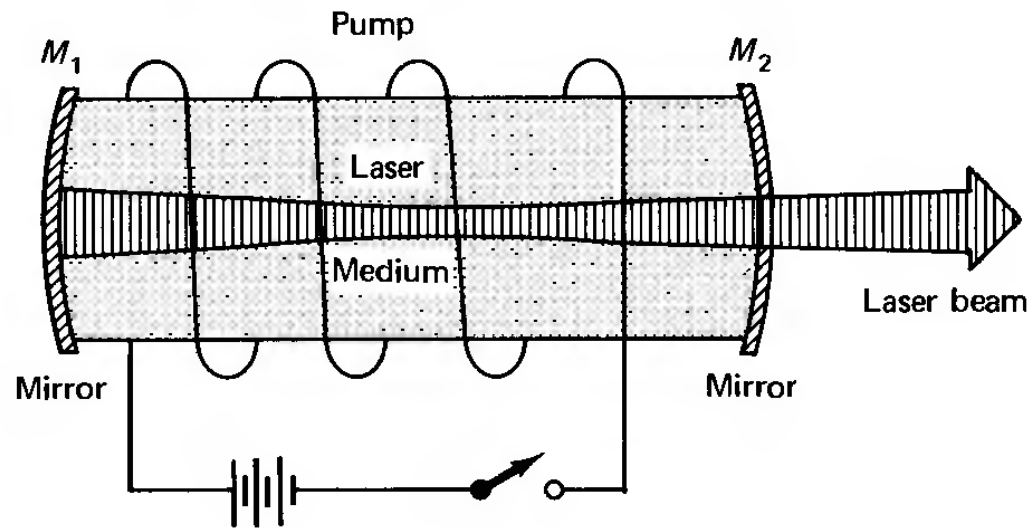
$$\approx 1.3 \times 10^{-38}.$$

(b) For the conditions of (a), at what temperature would the ratio N_x/N_0 be 1/2?

$$N_x/N_0 = 1/2.$$

$$T = \frac{E_x - E_0}{k(\ln 2)} = \frac{2.26 \text{ eV}}{(8.62 \times 10^{-5} \text{ eV/K})(\ln 2)}$$

$$= 38\,000 \text{ K.} \qquad (\text{Ans'}$$

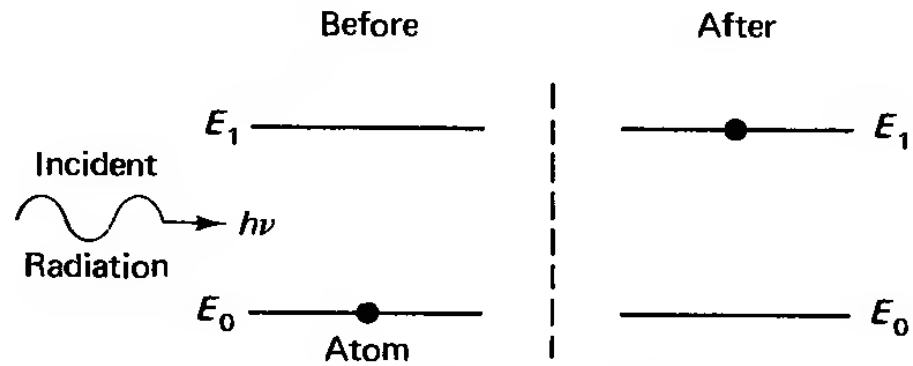


Laser Basics

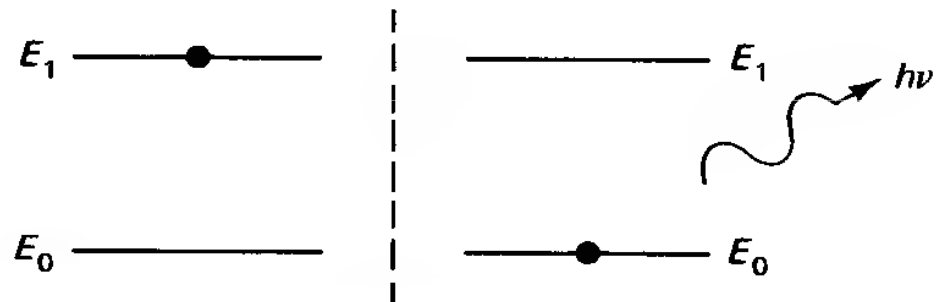
The word ***laser*** is an acronym that stands for ***Light Amplification by the Stimulated Emission of Radiation***.

EINSTEIN'S QUANTUM THEORY OF RADIATION

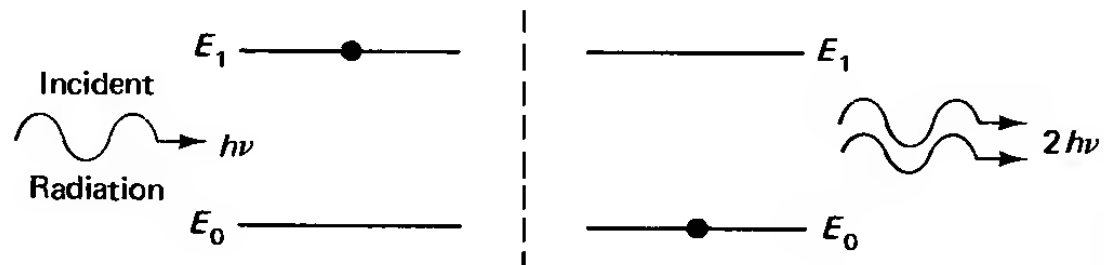
Einstein *A* and *B* Coefficients.



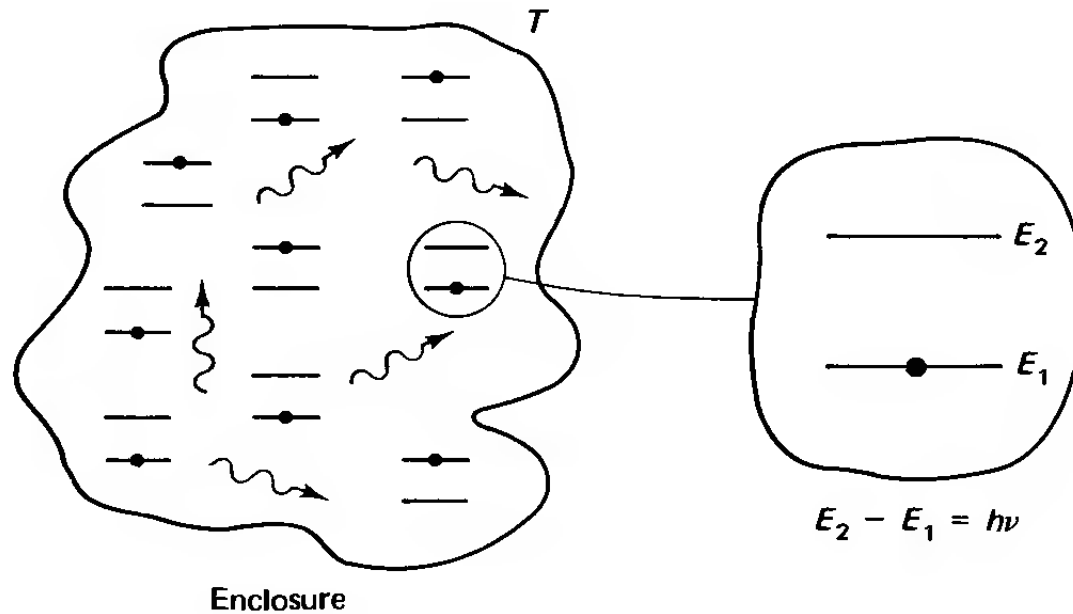
(a) Stimulated absorption



(b) Spontaneous emission



(c) Stimulated emission



Einstein A and B Coefficients.

Spontaneous Emission (A_{21}).

$$\left(\frac{dN_2}{dt}\right)_{\text{spont}} = -A_{21} N_2$$

$$N_2(t) = N_{20} e^{-A_{21}t}$$

Stimulated Emission (B_{21}).

$$\left(\frac{dN_2}{dt}\right)_{se} = -B_{21} N_2 \rho(\nu)$$

Absorption (B_{12}).

$$\left(\frac{dN_1}{dt}\right)_{\text{abs}} = -B_{12} N_1 \rho(\nu)$$

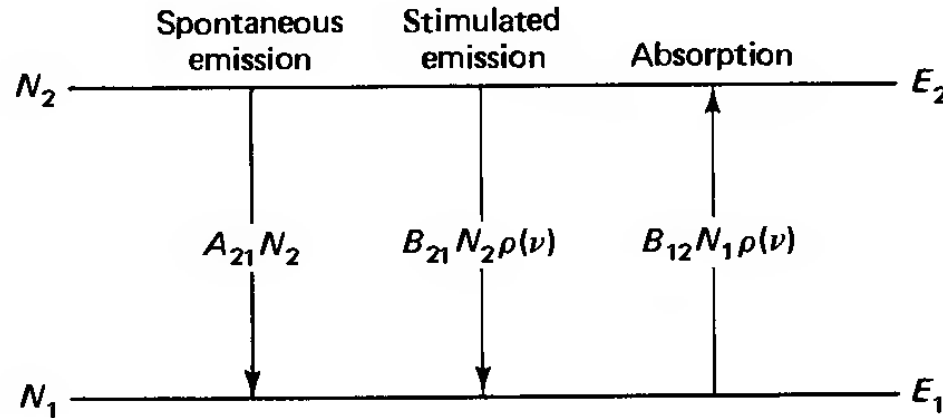


Figure 21-3 Radiative processes that affect the number of atoms at energy E_1 and E_2 . The two emission processes remove atoms from level E_2 and add them to level E_1 . The absorption process involves transitions from E_1 to E_2 .

Einstein's assumptions and their implications:

1. Thermodynamic equilibrium at arbitrary temperature T exists between the radiation field and the atoms.
2. The radiation field $\rho(\nu)$ has the spectral distribution characteristic of a black-body at temperature T .
3. The atom population densities N_1 and N_2 at energy levels E_1 and E_2 , respectively, are distributed according to the Boltzmann distribution at that temperature.
4. Population densities N_1 and N_2 are constant in time.

$$\frac{dN_2}{dt} = 0 = -N_2 A_{21} - N_2 B_{21} \rho(\nu) + N_1 B_{12} \rho(\nu) \quad \frac{A_{21}}{B_{12} e^{h\nu/kT} - B_{21}} = \frac{8\pi h \nu^3}{c^3} \frac{1}{e^{h\nu/kT} - 1}$$

$$\rho(\nu) = \frac{8\pi h \nu^3}{c^3} \frac{1}{e^{h\nu/kT} - 1} \quad \left(\frac{A_{21}}{B_{21}} - \frac{8\pi h \nu^3}{c^3} \frac{B_{12}}{B_{21}} \right) e^{h\nu/kT} - \left(\frac{A_{21}}{B_{21}} - \frac{8\pi h \nu^3}{c^3} \right) = 0$$

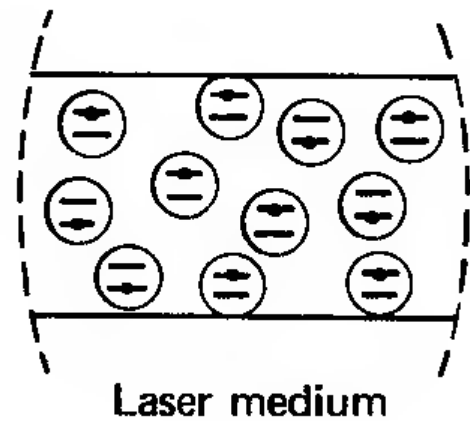
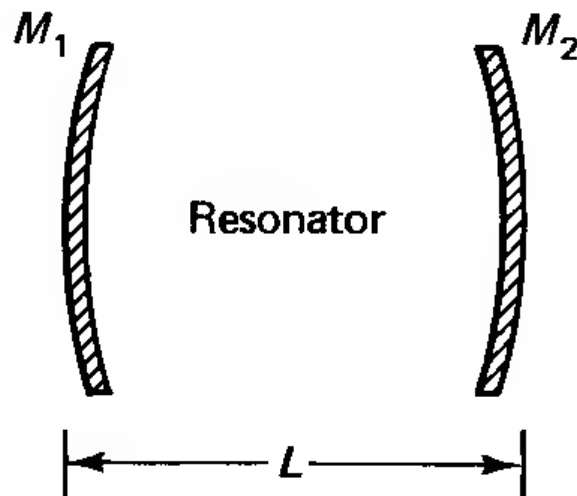
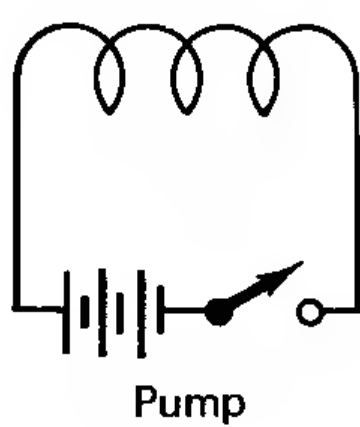
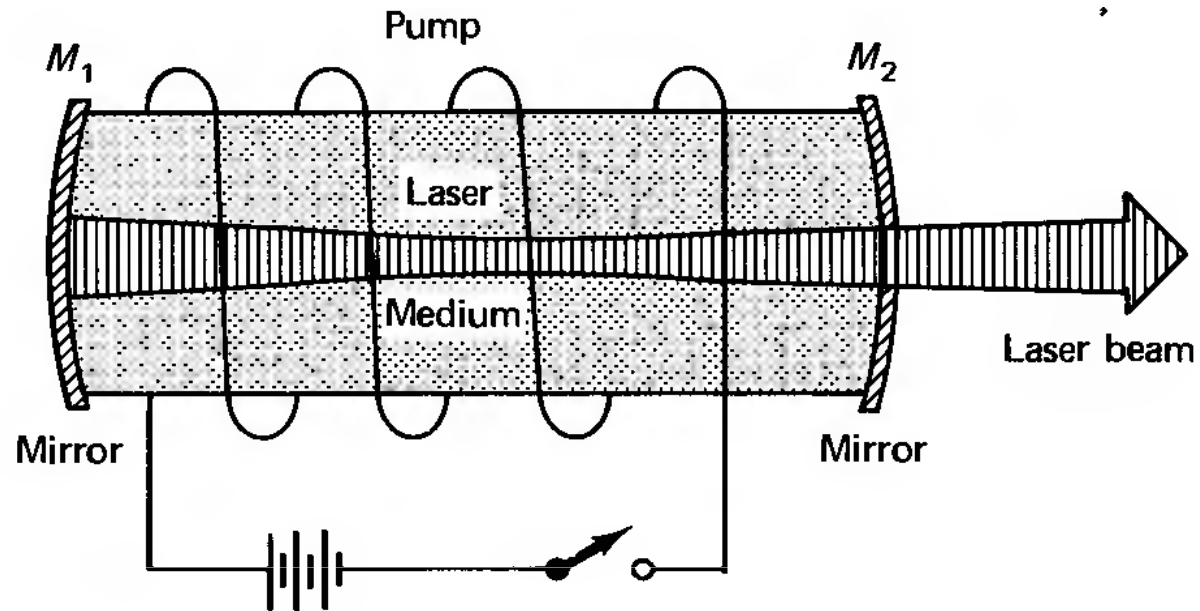
$$\frac{N_2}{N_1} = e^{-(E_2 - E_1)/kT} = e^{-h\nu/kT}$$

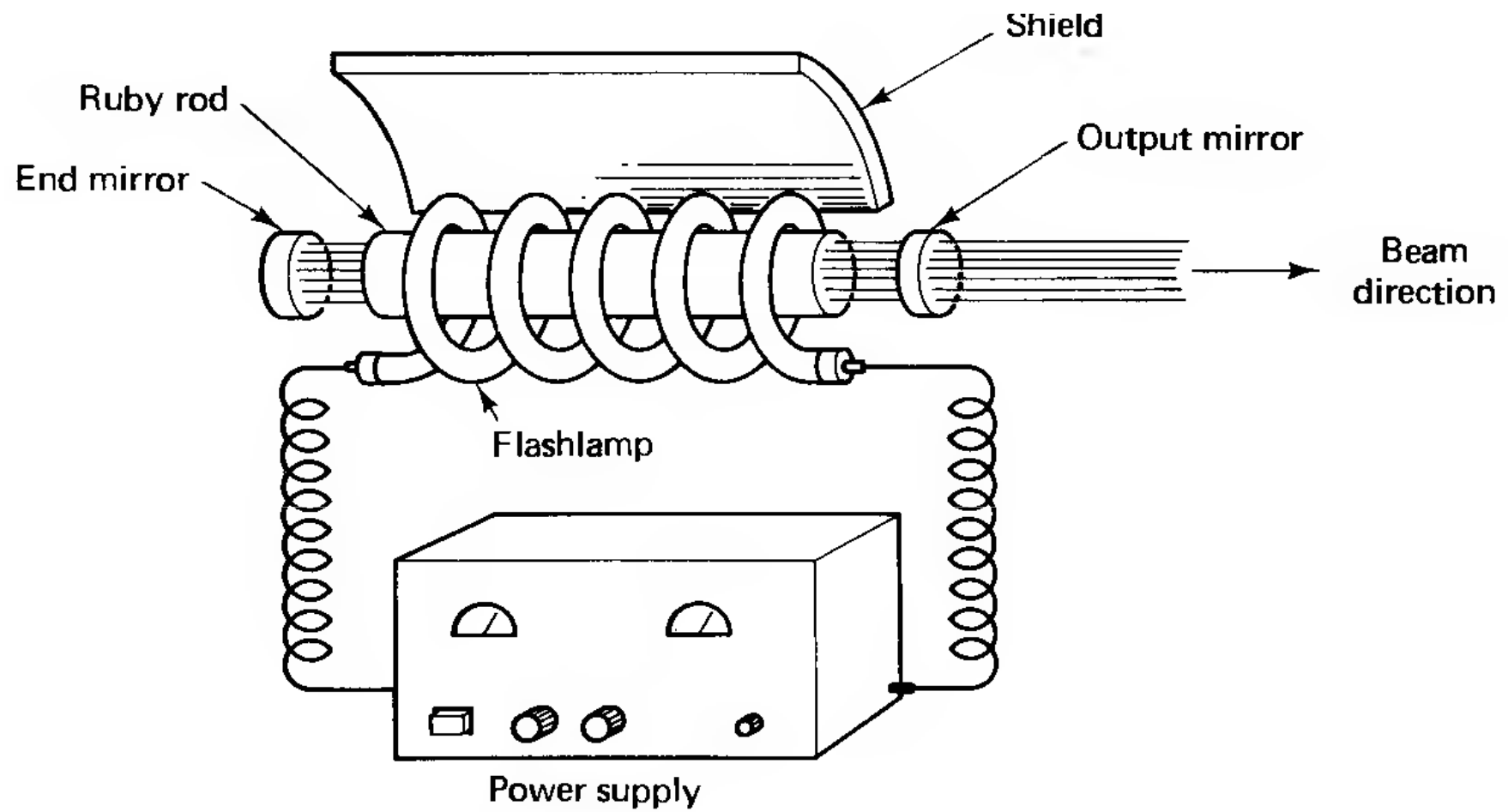
$$\frac{A_{21}}{B_{21}} = 8\pi h \nu^3 / c^3$$

$$\rho(\nu) = \frac{A_{21}}{B_{12}(N_1/N_2) - B_{21}} = \frac{A_{21}}{B_{12} e^{h\nu/kT} - B_{21}}$$

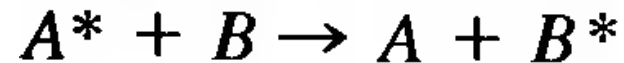
$$B_{12} = B_{21}$$

ESSENTIAL ELEMENTS OF A LASER





The pump is an external energy source that produces a population inversion in the laser medium. Pumps can be optical, electrical, chemical, or thermal in nature, so long as they provide energy that can be coupled into the laser medium to excite the atoms and create the required population inversion.



The most important requirement of the amplifying medium is its ability to support a population inversion between two energy levels of the laser atoms. This is accomplished by exciting (or pumping) more atoms into the higher energy level than exist in the lower level.

Boltzmann distribution, $N_2/N_1 = e^{-\Delta E/kT}$, where $\Delta E = E_2 - E_1$,

a resonator, an optical "feedback device" that directs photons back and forth through the laser (amplifying) medium.

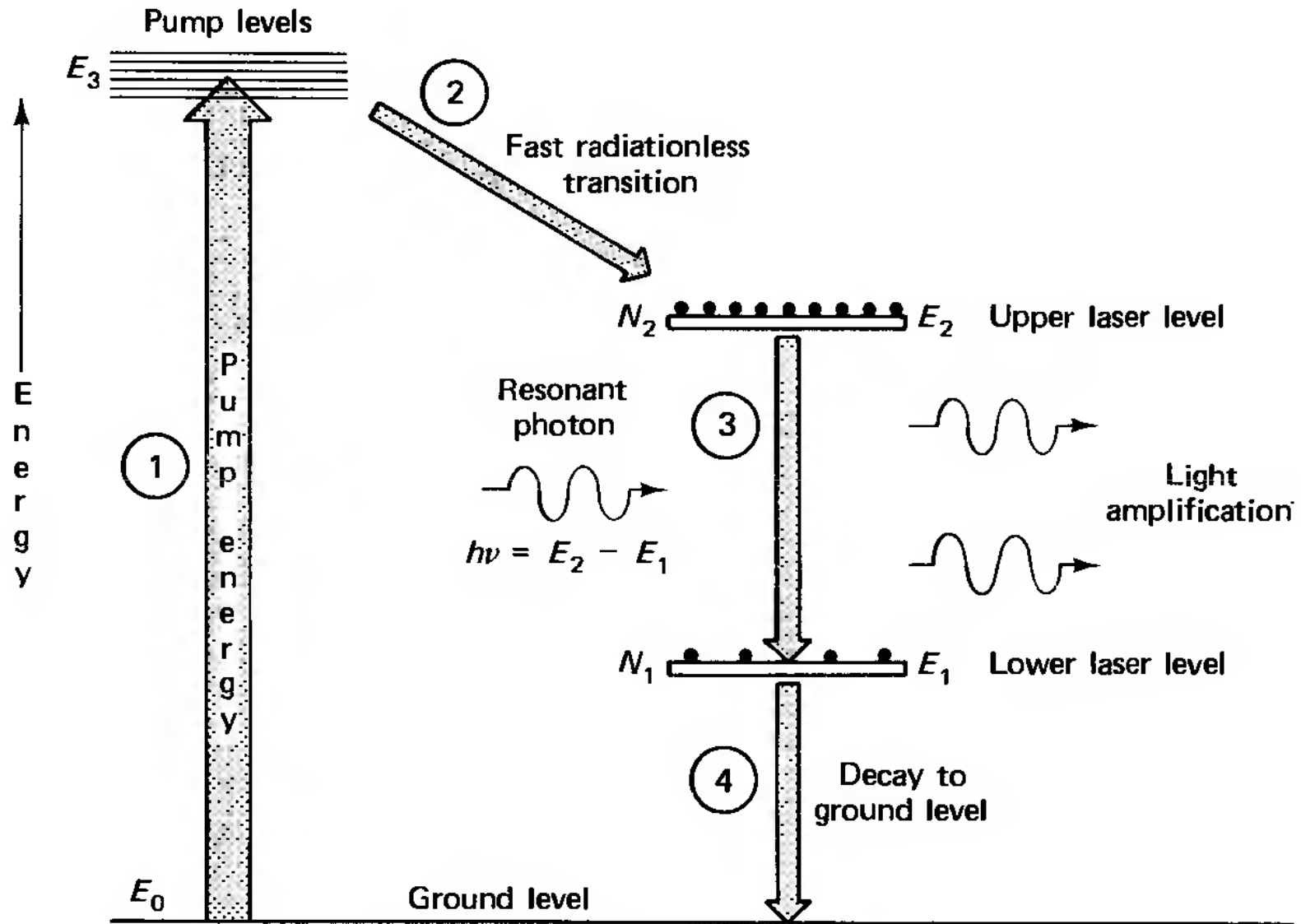
Fabry-Perot condition $t = L$, where $\theta_i = 0$

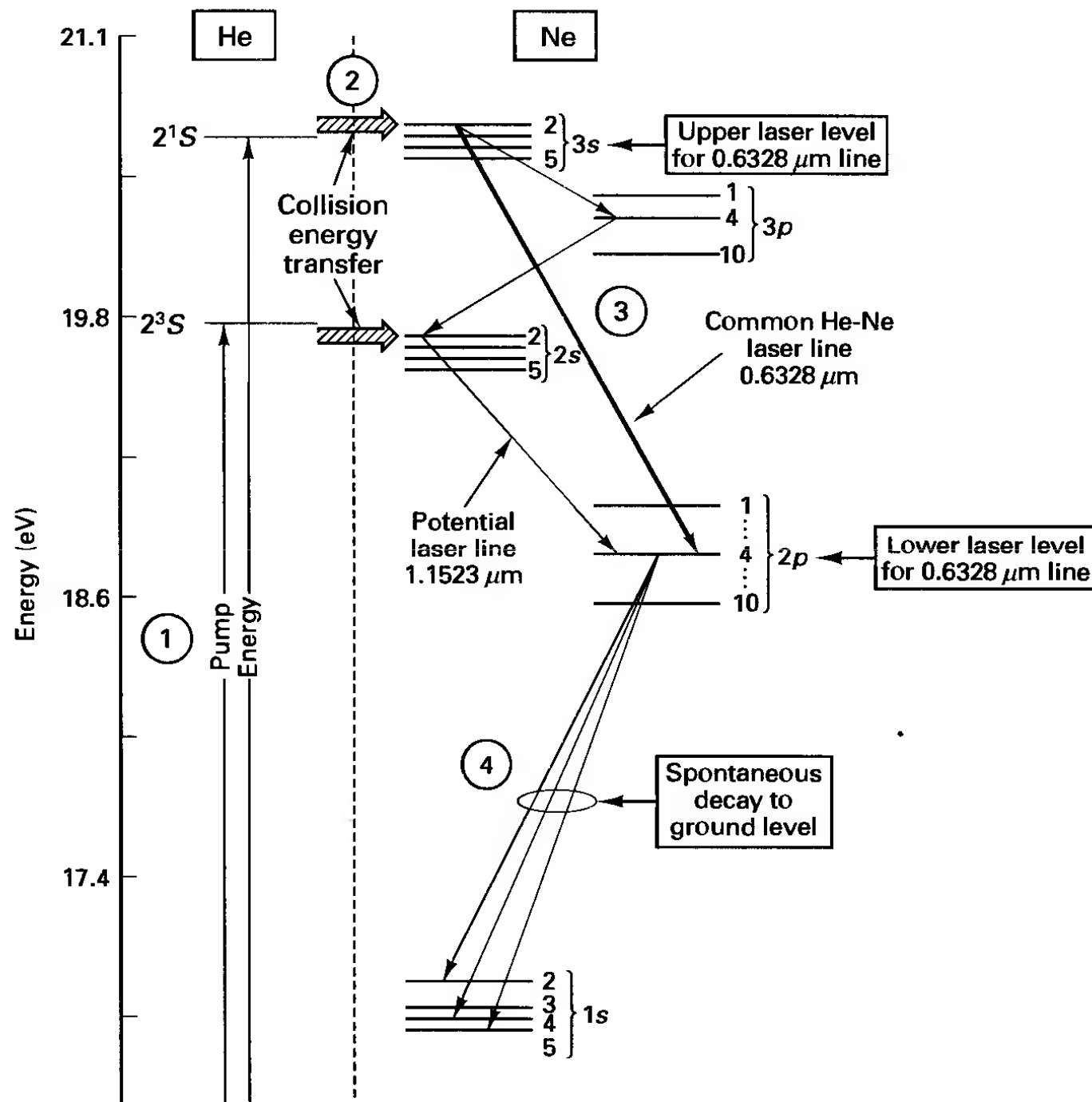
$$2t \cos \theta_i = m\lambda$$

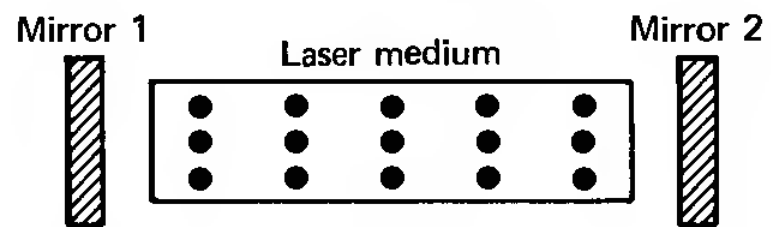
$$m\lambda/2 = L$$

standing waves in a string vibrating resonantly between two fixed ends.

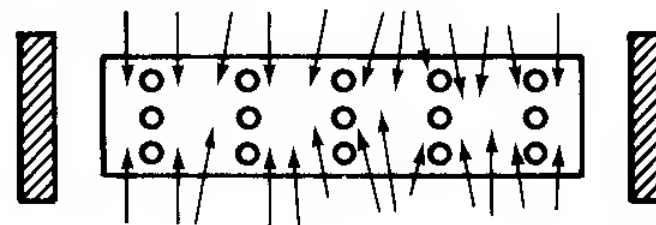
SIMPLIFIED DESCRIPTION OF LASER OPERATION



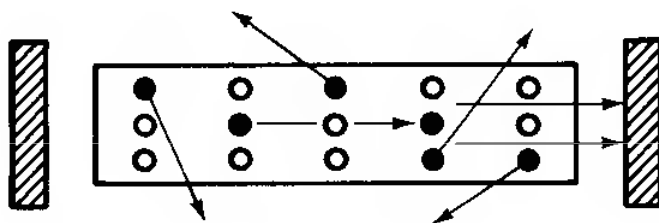




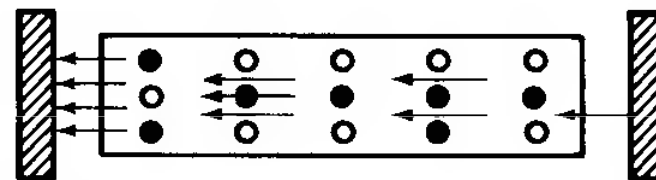
(a)



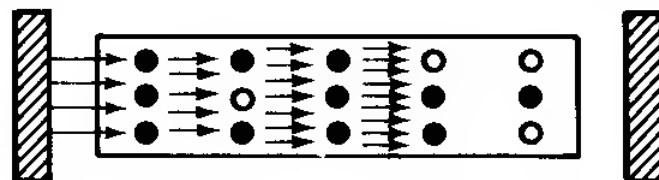
(b)



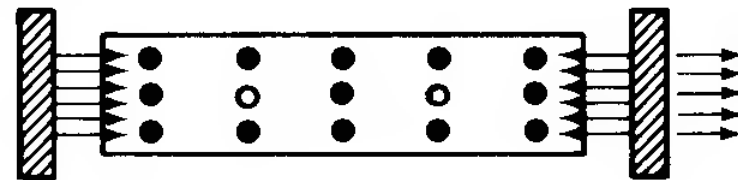
(c)



(d)



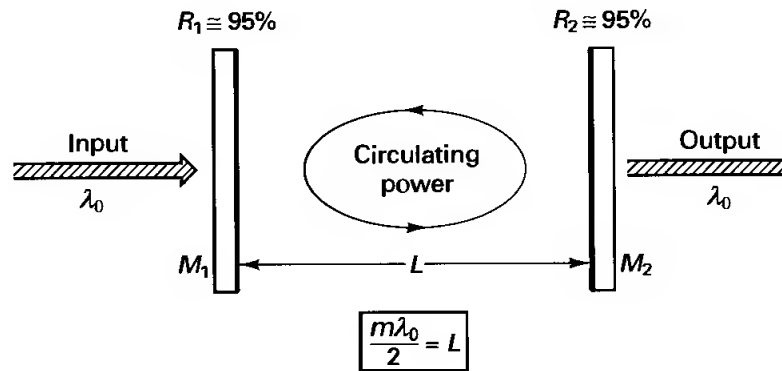
(e)



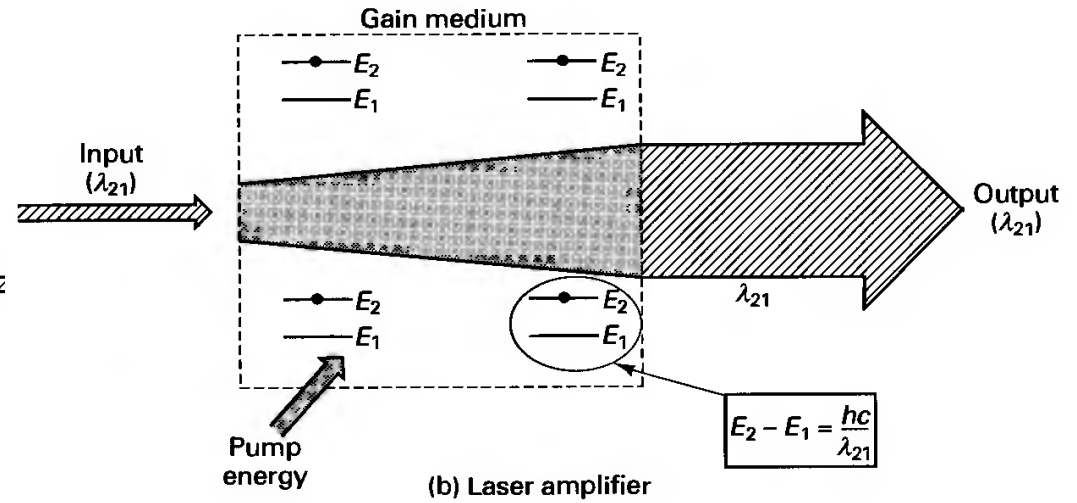
(f)

1. A *population inversion* between two appropriate energy levels in the laser medium. This is achieved by the pumping process and the existence of a metastable state.
2. *Seed photons* of proper energy and direction, coming from the ever-present spontaneous emission process between the two laser energy levels. These initiate the *stimulated emission* process.
3. An *optical cavity* that confines and directs the growing number of resonant energy photons back and forth through the laser medium, continually exploiting the population inversion to create more and more stimulated emissions, thereby creating more and more photons directed back and forth between the mirrors, and so on.
4. Coupling a certain fraction of the laser light wave (the cavity photon population) out of the cavity through the *output coupler mirror* to form the external laser beam.

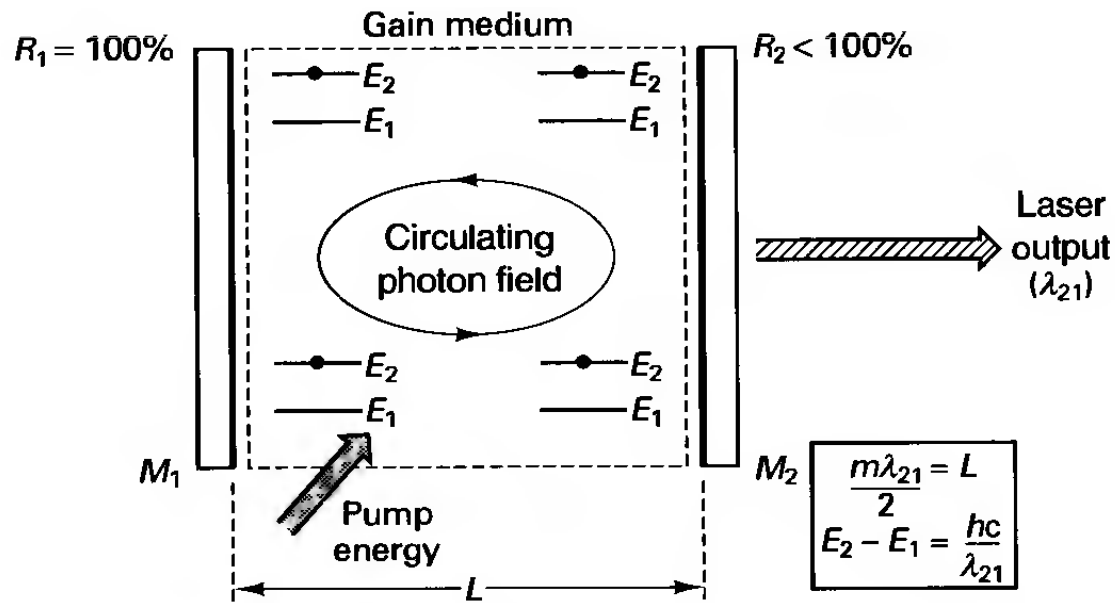
Comparing Fabry-Perot and Laser Resonators.



(a) Fabry-Perot cavity



(b) Laser amplifier



(c) Laser oscillator

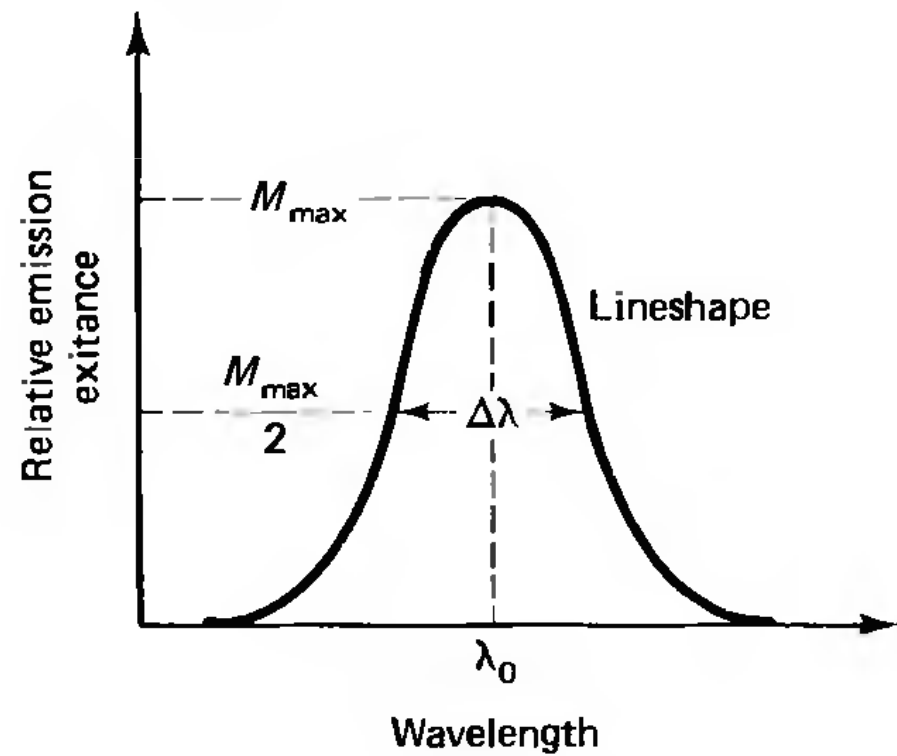
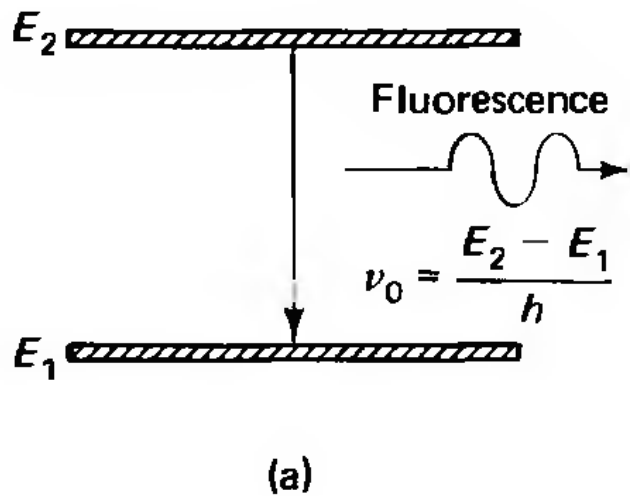
CHARACTERISTICS OF LASER LIGHT

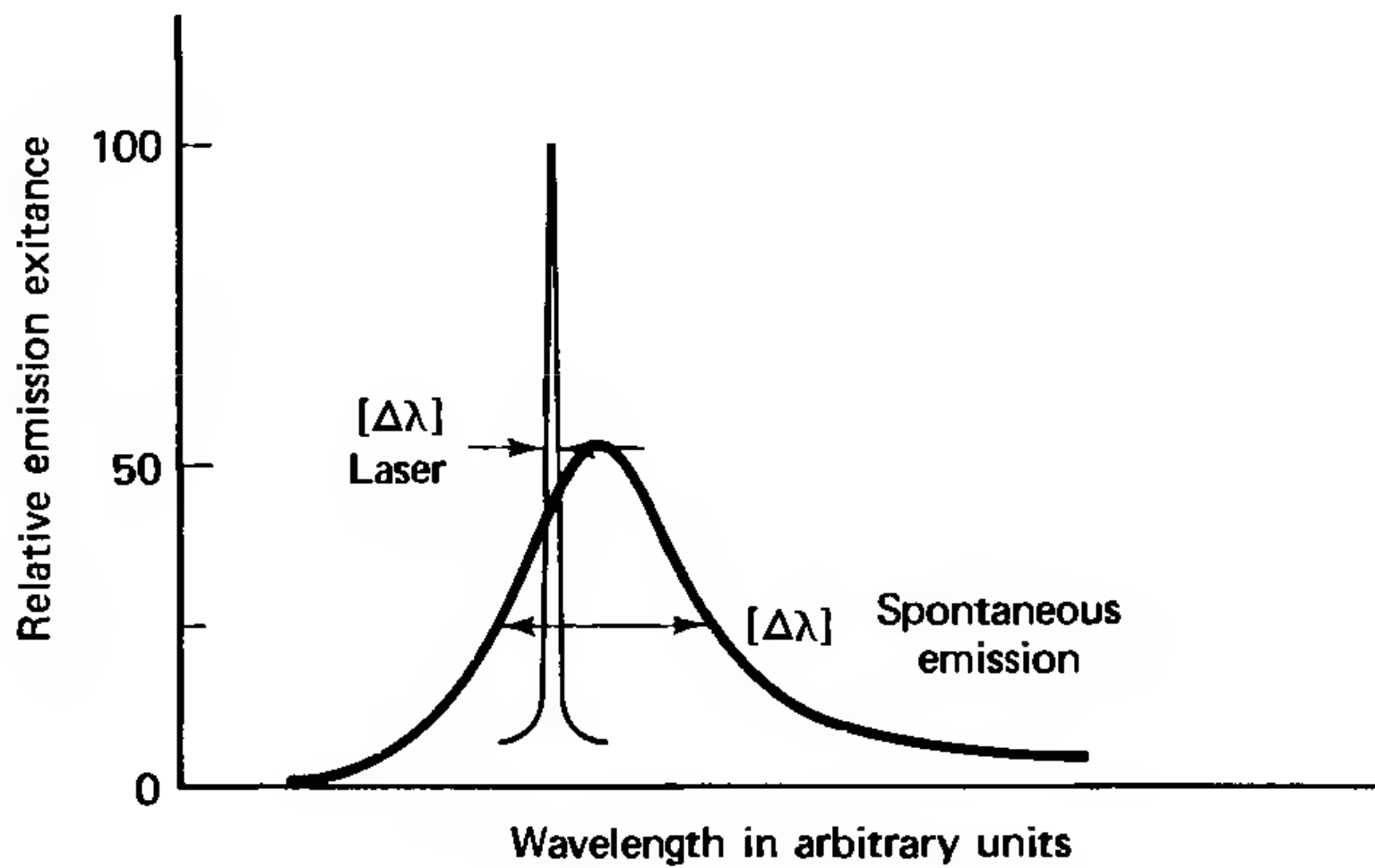
Monochromaticity.

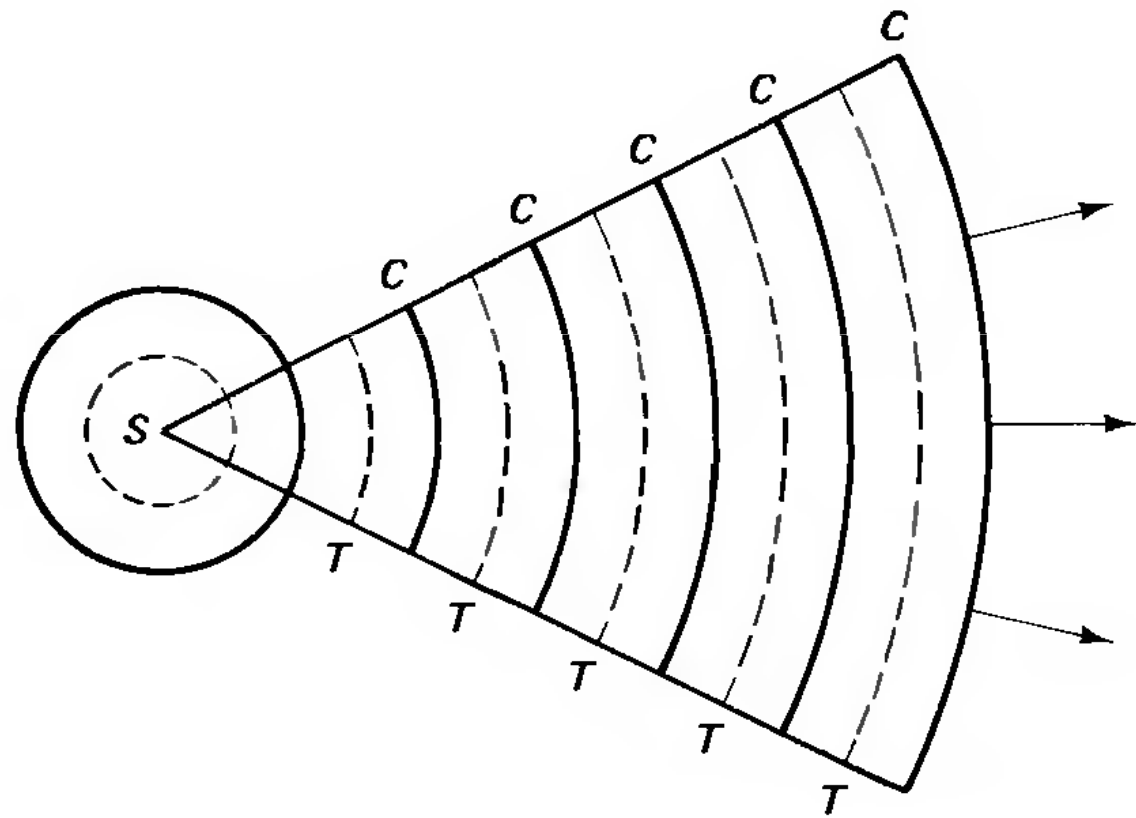
Coherence.

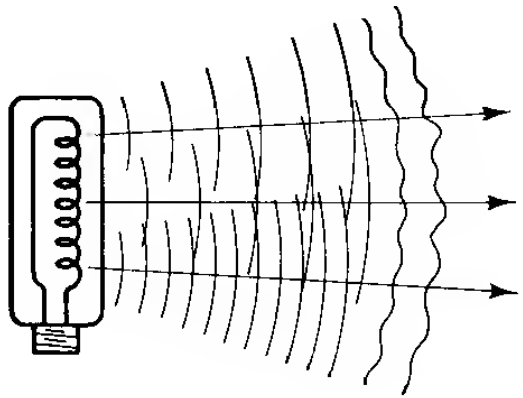
Directionality.

Focusability.

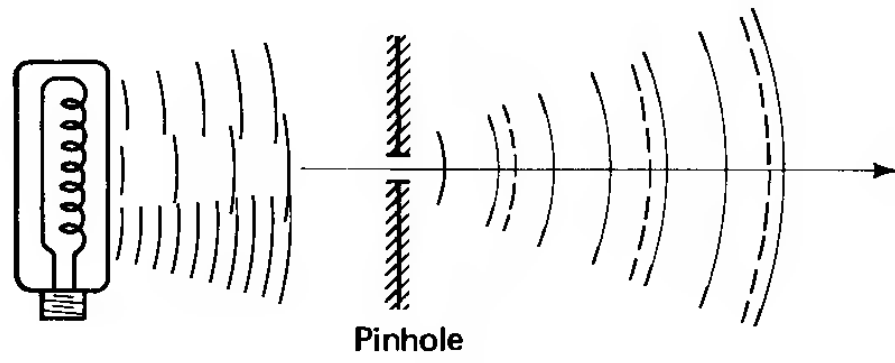




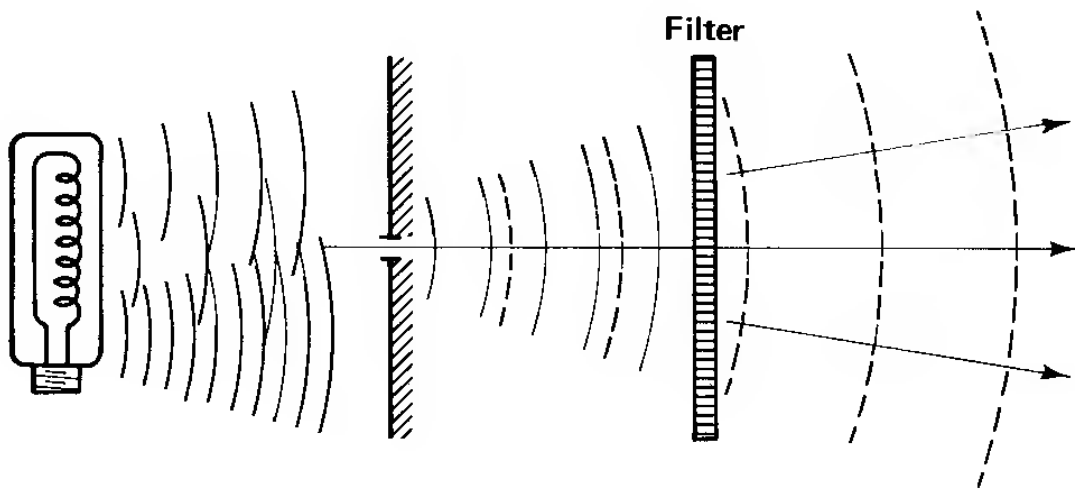




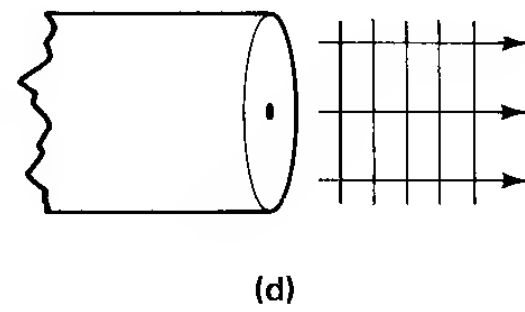
(a)



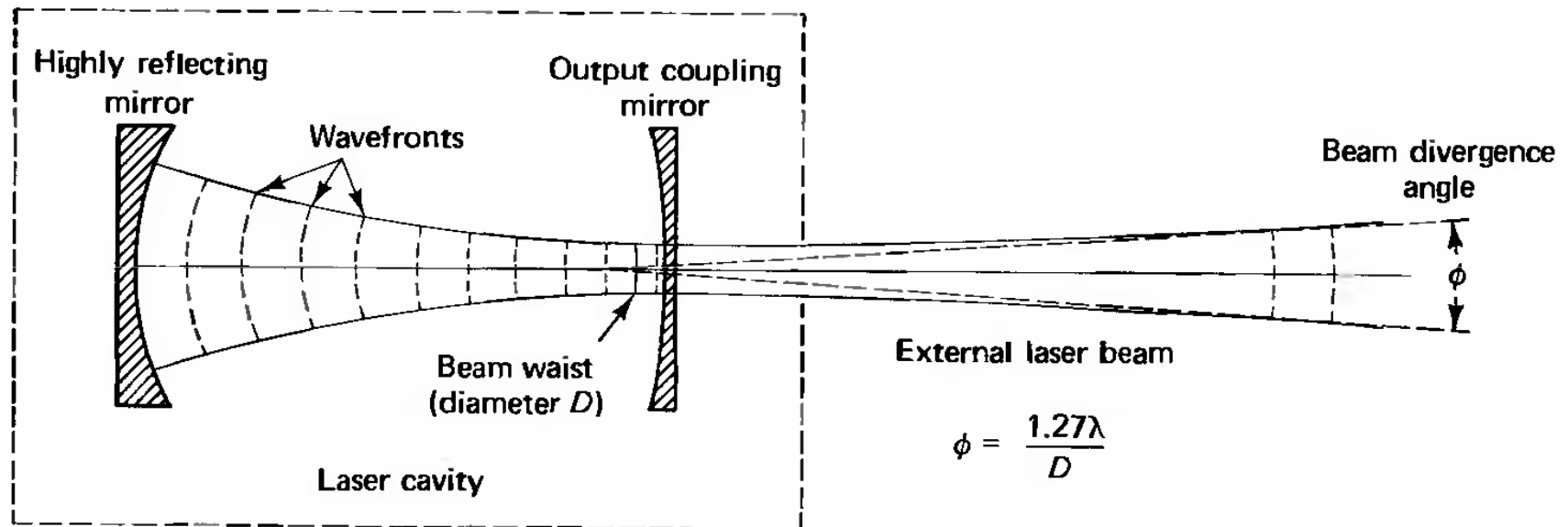
(b)



(c)



(d)



Example

(a) He-Ne lasers (632.8 nm) have an internal beam waist of diameter near 0.5 mm. Determine the beam divergence.

(b) Since we can control the beam waist D by laser cavity design and “select” the wavelength by choosing different laser media, what lower limit might we expect for the beam divergence? How directional can lasers be? Suppose we design a laser with a beam waist of 0.5-cm diameter and a wavelength of 200 nm. By what factor is the beam divergence improved?

Solution (a)

$$\phi = \frac{1.27\lambda}{D} = \frac{(1.27)(632.8 \times 10^{-9} \text{ m})}{5 \times 10^{-4} \text{ m}} = 1.6 \times 10^{-3} \text{ rad}$$

This is a typical laser-beam divergence, indicating that the beam width increases about 1.6 cm every 1000 cm.

(b)

$$\phi = \frac{1.27\lambda}{D} = \frac{(1.27)(200 \times 10^{-9} \text{ m})}{5 \times 10^{-3} \text{ m}} = 5.1 \times 10^{-5} \text{ rad}$$

Thus the beam divergence angle becomes about 5×10^{-5} rad, roughly a 30-fold decrease in beam spread over the He-Ne laser described in part (a). This beam would spread about 1.6 cm every 320 m. Clearly, if beam waist size is at our command and lasers can be built with wavelengths below the ultraviolet, there is no limit to how parallel and directional the laser beam can be made.

Example

(a) Small gas lasers typically have power outputs P of 1 mW. Neodymium-glass lasers, such as those under development for the production of laser-induced fusion, boast of power outputs near 10^{14} W! Using these two extremes and an average energy of 10^{-19} J per visible photon ($E = h\nu$), determine the approximate range of photon output from lasers.

(b) For comparison, consider a broadband thermal source with a radiating surface equal to that of the beam waist of a 1-mW He-Ne laser with diameter of 0.5 mm (or area of $A = 2 \times 10^{-7}$ m²). Let the surface emit radiation at a wavelength of 633 nm with a linewidth of $\Delta\lambda = 100$ nm and temperature $T = 1000$ K. Determine the photon output rate of this thermal source.

Solution (a) The rate of photon output is given by $P/h\nu$. Thus

$$\frac{P}{h\nu} = \frac{10^{-3} \text{ J/s}}{10^{-19} \text{ J/photon}} = 10^{16} \text{ photons/s} \quad .$$

$$\frac{P}{h\nu} = \frac{10^{14} \text{ J/s}}{10^{-19} \text{ J/photon}} = 10^{33} \text{ photons/s}$$

(b) The photon output rate for the broadband source can be calculated from the equation³

$$\text{thermal photons/s} \cong \frac{1}{\lambda^2} \frac{1}{e^{h\nu/kT} - 1} \Delta A \Delta \nu \quad (21-11)$$

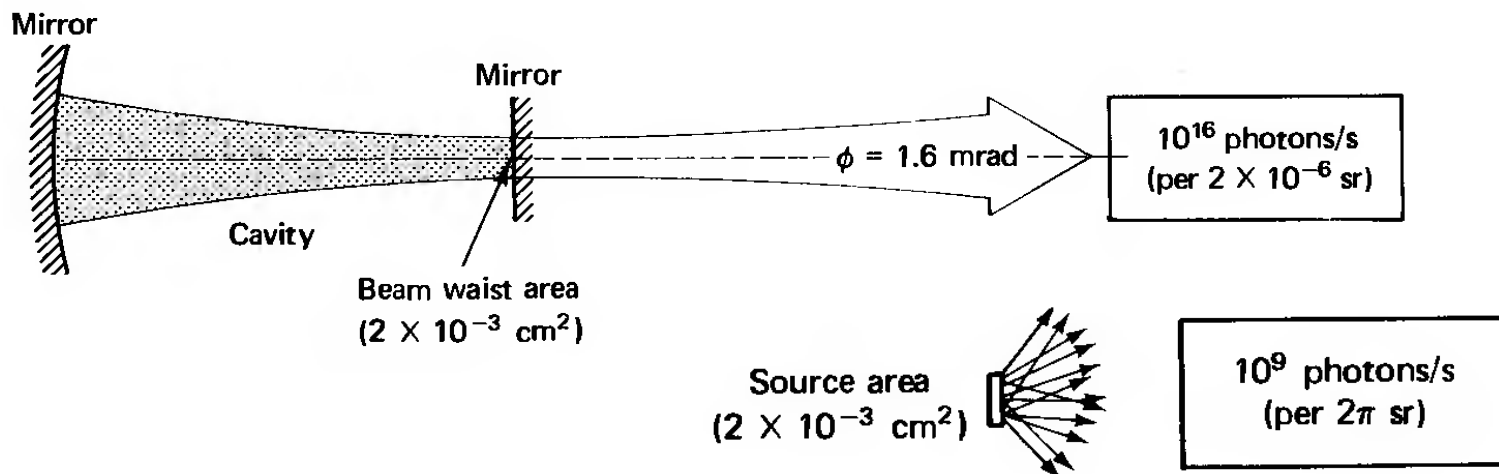
The frequency $\nu = c/\lambda = 4.74 \times 10^{14}$ Hz. The frequency bandwidth $\Delta\nu = (c/\lambda^2)\Delta\lambda = 7.5 \times 10^{13}$ Hz. The argument of the exponent is

$$\frac{h\nu}{kT} = \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(4.74 \times 10^{14} \text{ s}^{-1})}{(1.38 \times 10^{-23} \text{ J/K})(1000 \text{ K})} = 22.77$$

Substituting all values into Eq. (21-11), the thermal photon output per second is

$$\left(\frac{1}{(633 \times 10^{-9})^2 \text{ m}^2}\right) \left(\frac{1}{7.76 \times 10^9}\right) (2 \times 10^{-7} \text{ m}^2) (7.5 \times 10^{13} \text{ s}^{-1}) \cong 5 \times 10^9$$

We find a rate of only about 10^9 photons/s! This value is seven orders of magnitude smaller than the photon output rate of a low-power 1-mW He-Ne laser and 24 orders of magnitude smaller than a powerful neodymium-glass laser. The comparison is summarized in Figure 21-16.



We see also from Figure 21-16 that the He-Ne laser emits 10^{16} photons/s into a very small solid angle of about 2×10^{-6} sr, whereas the thermal emitter, acting as a Lambertian source, radiates 10^9 photons/s into a forward, hemispherical solid angle of 2π sr. If we were to ask how many thermal photons/second are emitted by the thermal source into a solid angle equal to that of the laser, we would find the answer to be a mere 320 photons/s:

$$(10^9 \text{ photons/s}) \left(\frac{2 \times 10^{-6} \text{ sr}}{6.28 \text{ sr}} \right) = 320 \text{ photons/s}$$

The contrast between 10^{16} photons/s for the laser source and 320 photons/s for the thermal source is now even more dramatic.

Carrying our comparison of source intensity between laser and nonlaser sources one step further, we can determine how the radiance [$\text{W}/(\text{cm}^2\text{-sr})$] of a high-power, nonlaser source compares with that of a low-power laser.

Example

We choose for the nonlaser source a super-high-pressure mercury lamp, capable of a source radiance L_e equal to $250 \text{ W}/(\text{cm}^2\text{-sr})$. Such lamps were about the best high-radiance sources available before the advent of the laser. For the laser source, we choose a 4-mW He-Ne laser with beam-waist diameter of 0.5 mm and beam divergence angle of 1.6 mrad, emitting at a wavelength of 633 nm. The geometry is shown in Figure 21-17. It should be clear from the figure that the laser can be considered a radiating surface of area equal to that of the beam waist. The radiant power of the surface is equal to that of the laser, and the surface radiates only into the solid angle dictated by the laser.

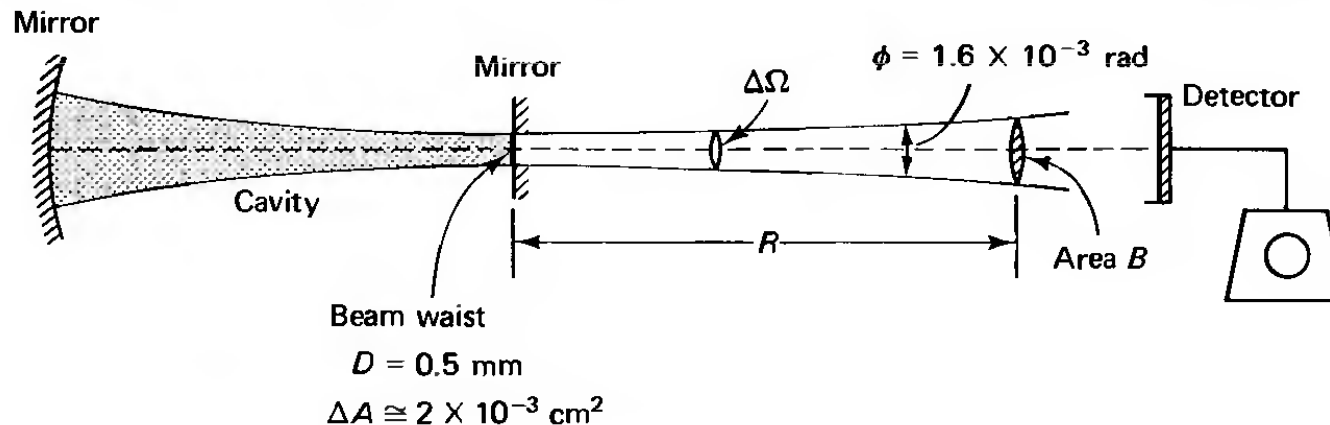


Figure 21-17 Radiance of a low-power He-Ne laser. The radiance of the laser (beam waist) “seen” by the detector is about $10^6 \text{ W}/\text{cm}^2\text{-sr}$.

Solution Using the definition for radiance L_e in terms of laser power Φ_e , source or beam-waist area A , and solid angle $\Delta\Omega$,

$$L_e = \frac{\Phi_e}{\Delta A \Delta\Omega} \quad (21-12)$$

the radiance of the laser is found to be 10^6 W/cm²-sr, as follows: Referring to Figure 21-17, the solid angle $\Delta\Omega = \text{area } B/R^2$, where

$$\text{area } B = \pi \left[R \tan \left(\frac{\phi}{2} \right) \right]^2 \cong \frac{\pi R^2 \phi^2}{4}$$

Then substituting into Eq. (21-12), the radiance is

$$L_e = \frac{4\Phi_e}{\pi\phi^2 \Delta A} = \frac{4(4 \times 10^{-3} \text{ W})}{\pi (1.6 \times 10^{-3})^2 (2 \times 10^{-3} \text{ cm}^2)} = 1 \times 10^6 \text{ W/cm}^2\text{-sr}$$

Again, the comparison between laser source and high-intensity mercury lamp is dramatic. We conclude that where brightness and radiance are important in the selection of light sources, the laser stands alone.

2. Assume that an atom has two energy levels separated by an energy corresponding to a wavelength of 632.8 nm, as in the He-Ne laser. If we suppose that all the atoms are located in one or the other of these two states, what fraction of atoms is in the upper state at room temperature, $T = 300$ K, according to the Boltzmann distribution?

$$\mathbf{21-2.} \ N_2/N_1 = 1.2 \times 10^{-33}$$

4. Treating the sun as a blackbody radiator, determine its spectral energy density $\rho(\nu)$ in the visible region near $\lambda = 550$ nm. Assume the surface temperature of the sun to be about 6000 K.

21-4. $1.30 \times 10^{-15} \text{ J/m}^3\text{-Hz}$

6. Calculate the ratio of stimulated to spontaneous transitions for green light at $0.5 \mu\text{m}$ within a dense blackbody plasma at a temperature of 5000 K. That is, determine the value of the ratio

$$\frac{(dN/dt)_{se}}{(dN/dt)_{sp}} = \frac{-B_{21} N_2 \rho(\nu)}{-A_{21} N_2}, \quad \text{where} \quad \frac{B_{21}}{A_{21}} = \frac{\lambda^3}{8\pi h}$$

What does the numerical value of this ratio imply?

6. 0.00318; spontaneous emission is about 314 times larger than stimulated emission. This is to be expected since the ratio calculated is greater than 1 whenever $h\nu \gg kT$.

8. Suppose that the coherence time of a light beam is roughly equal to the reciprocal of its frequency linewidth (FWHM). What then is the coherence time and coherence length of the He-Ne laser in Table 21-1?

21-8. 0.13 ms; 40 km

9. A He-Ne laser has a beam waist (diameter) equal to about 1 mm. What is its beam-spread angle in the far field?

21-9. 0.80 mrad

- 12.** With reference to the beam expander shown in Figure 21-20, suppose $f_2/f_1 = 10$ and the beam divergence of the incident beam is 1 mrad.
- (a) What is the beam divergence of the expanded beam?
 - (b) If the expanded beam is focused by a subsequent lens of power 10 diopters, what is the beam waist at the focal spot?
 - (c) If the power in the incident beam is 1 mW, what is the irradiance (W/m^2) at the focal spot?

21-12. (a) 10^{-4} rad (b) $10 \mu\text{m}$ (c) 12.7 MW/m^2

21-13. (a) 1 W (b) 20 W

21-14. (a) $0.81 \mu\text{m}$; $0.75 \mu\text{m}$; $0.585 \mu\text{m}$; $0.525 \mu\text{m}$ (b) 76%; 70%; 55%; 49%

- 13.** Refer to Figure 21-8a. Assume that 1 W of incident laser power of resonant wavelength λ_0 is incident on the Fabry-Perot cavity.
- (a)** What is the output power?
 - (b)** How much power is circulating within the cavity?
 - (c)** Describe what must be happening at mirror M_1 to make your answers to (a) and (b) valid.

21-12. (a) 10^{-4} rad (b) $10\ \mu\text{m}$ (c) $12.7\ \text{MW/m}^2$

21-13. (a) 1 W (b) 20 W

21-14. (a) $0.81\ \mu\text{m}$; $0.75\ \mu\text{m}$; $0.585\ \mu\text{m}$; $0.525\ \mu\text{m}$ (b) 76%; 70%; 55%; 49%

14. For a Nd:YAG laser, there are four pump levels located at 1.53 eV, 1.653 eV, 2.119 eV, and 2.361 eV above the ground state energy level.
- (a) What is the wavelength associated with the photon energy required to populate each of the pump levels?
 - (b) Knowing that a Nd:YAG laser emits photons of wavelength $1.064 \mu\text{m}$, determine the *quantum efficiency* associated with each of the four pump levels.

21-12. (a) 10^{-4} rad (b) $10 \mu\text{m}$ (c) 12.7 MW/m^2

21-13. (a) 1 W (b) 20 W

21-14. (a) $0.81 \mu\text{m}$; $0.75 \mu\text{m}$; $0.585 \mu\text{m}$; $0.525 \mu\text{m}$ (b) 76%; 70%; 55%; 49%

- 21-2.** $N_2/N_1 = 1.2 \times 10^{-33}$
- 21-4.** $1.30 \times 10^{-15} \text{ J/m}^3\text{-Hz}$
- 21-6.** 0.00318; spontaneous emission is about 314 times larger than stimulated emission.
This is to be expected since the ratio calculated is greater than 1 whenever $h\nu \gg kT$.
- 21-7.** (a) See Table 21-1. (b) Linewidth is about one ten-millionth as wide.
- 21-8.** 0.13 ms; 40 km
- 21-9.** 0.80 mrad
- 21-11.** (b) $2.5 \times 10^5 \text{ W/m}^2\text{-sr-Hz}$
- 21-12.** (a) 10^{-4} rad (b) $10 \mu\text{m}$ (c) 12.7 MW/m^2
- 21-13.** (a) 1 W (b) 20 W
- 21-14.** (a) $0.81 \mu\text{m}$; $0.75 \mu\text{m}$; $0.585 \mu\text{m}$; $0.525 \mu\text{m}$ (b) 76%; 70%; 55%; 49%
- 21-15.** (a) 31.5 W (b) 1.26%

6-1 The Lyman series in the line spectra of atomic hydrogen is the name for the light emitted from transitions from excited states to the $n = 1$ hydrogen ground state. The Balmer series refers to the light emitted from transitions from excited states with $n \geq 3$ to the $n = 2$ energy state.

- a.** Find the wavelengths of the three shortest-wavelength photons in the Lyman series. In what range of the electromagnetic spectrum are the spectral lines of the Lyman series?
- b.** Find the wavelengths of the three shortest-wavelength photons in the Balmer series. In what range of the electromagnetic spectrum are the spectral lines of the Balmer series?

6-1. The allowed energies of the electron in the hydrogen atom are $E_n = - (13.6 \text{ eV})/n^2$, $n = 1, 2, 3, \dots$. The energies required for this problem are

$$E_1 = -13.6 \text{ eV} \quad E_2 = -3.40 \text{ eV} \quad E_3 = -1.51 \text{ eV} \quad E_4 = -0.850 \text{ eV} \quad E_5 = -0.544 \text{ eV}$$

The wavelength associated with a transition is

$$\lambda_{mn} = \frac{hc}{E_m - E_n} = \frac{1240 \text{ eV} \cdot \text{nm}}{E_m - E_n}$$

(a) Lyman Series:

$$\lambda_{21} = \frac{1240}{-3.4 - (-13.6)} \text{ nm} = 122 \text{ nm} \quad \lambda_{31} = \frac{1240}{-1.51 - (-13.6)} \text{ nm} = 103 \text{ nm} \quad \lambda_{41} = 97.3 \text{ nm}$$

These are in the ultraviolet (shorter than visible).

(b) Balmer Series:

$$\lambda_{32} = \frac{1240}{-1.51 - (-3.4)} \text{ nm} = 656 \text{ nm} \quad \lambda_{42} = \frac{1240}{-0.85 - (-3.4)} \text{ nm} = 486 \text{ nm} \quad \lambda_{52} = 434 \text{ nm}$$

These are in the visible range.

- 6-2 a. Will a photon of energy 5 eV likely be absorbed by a hydrogen atom originally in its ground state?
- b. What is the range of photon wavelengths that could ionize a hydrogen atom that is originally in its ground state?
- c. What is the range of photon wavelengths that could ionize a hydrogen atom that is originally in its $n = 2$ energy state?

6.2. (a) From the solution to problem 6-1, it is evident that an energy of 5 eV does not correspond to the energy difference between the ground state of hydrogen and an excited state of hydrogen. Therefore, since the photon must give up its energy in one lump, the photon will not be absorbed by the hydrogen atom. (Neglecting very unlikely nonlinear processes, of course.)

(b) An electron in hydrogen is ionized (leaves the proton) when its energy exceeds zero. Thus the energy of the photon must exceed $E_{\text{final}} - E_{\text{initial}} = 0 - (-13.6 \text{ eV}) = 13.6 \text{ eV}$. Any energy greater than this is possible since the free electron can have any energy. The corresponding wavelength range is

$$\lambda < \frac{hc}{13.6 \text{ eV}} = \frac{1240 \text{ eV} \cdot \text{nm}}{13.6 \text{ eV}} = 91.2 \text{ nm}$$

(c). In this case, the photon energy must exceed $0 - E_2 = 0 - (-3.4 \text{ eV}) = 3.4 \text{ eV}$. The wavelength range is then

$$\lambda < \frac{hc}{3.4 \text{ eV}} = \frac{1240 \text{ eV} \cdot \text{nm}}{3.4 \text{ eV}} = 365 \text{ nm}$$

- 6-3** The allowed rotational energies E_l^{rot} of a diatomic molecule are given by

$$E_l = \frac{l(l+1)\hbar^2}{2I}$$

In this expression l is the rotational quantum number and can take the values $l = 0, 1, 2, \dots$; I is the rotational inertia of the molecule about an axis through its center of mass; and $\hbar = h/2\pi$. The equilibrium separation of the two atoms in a diatomic hydrogen molecule H_2 is about 0.074 nm. The mass of each hydrogen atom is about 1.67×10^{-27} kg.

- a.** Show that the rotational inertia of the hydrogen molecule about an axis through its center of mass is about $I = 4.6 \times 10^{-48} \text{ kg} \cdot \text{m}^2$.
- b.** Find the difference in energy between the first excited rotational energy state and the ground rotational state. That is, find $E_1^{\text{rot}} - E_0^{\text{rot}}$. Express the answer in both J and eV.
- c.** Find the relative likelihood $P_{l=1}/P_{l=0}$ that a hydrogen molecule will be in its first excited rotational state in thermal equilibrium at room temperature, $T = 293 \text{ K}$. (Ignore possible state degeneracies.)

6-3. (a) The rotational inertia is

$$I = 2 m_H r^2 = 2(1.67 \times 10^{-27}) (0.074 \times 10^{-9}/2)^2 \text{ kg} \cdot \text{m}^2 = 4.6 \times 10^{-48} \text{ kg} \cdot \text{m}^2$$

$$(b) \Delta E = E_1^{\text{rot}} - E_0^{\text{rot}} = \hbar^2/I = (6.626 \times 10^{-34}/2\pi)^2/(4.6 \times 10^{-48}) \text{ J} = 2.4 \times 10^{-21} \text{ J} = 0.015 \text{ eV}$$

$$(c) P_1/P_0 = e^{-(\Delta E)/k_B T} = e^{-0.015/(8.62 \times 10^{-5} \cdot 293)} = 0.55$$

6-4 The allowed energies E_k^{vib} associated with the vibration of a diatomic molecule are given by

$$E_k^{\text{vib}} = (k + 1/2)hf$$

Here, k is the vibrational quantum number and can take the values $k = 0, 1, 2, \dots$ and f is the resonant frequency of the vibration. In a simple model of diatomic hydrogen H_2 , the resonant vibration frequency can be taken as $f = 1.3 \times 10^{14}$ Hz.

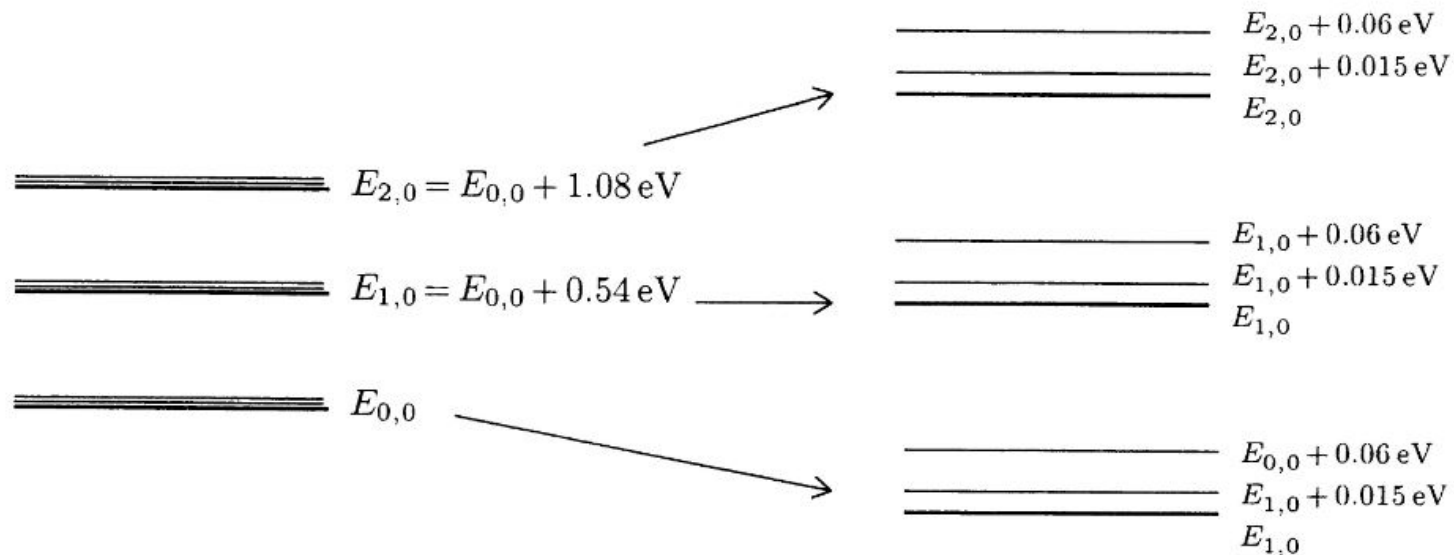
- a.** Find the difference in energy between the first excited vibrational energy state and the ground vibrational state of diatomic hydrogen. That is, find $E_1^{\text{vib}} - E_0^{\text{vib}}$. Express the answer in both J and eV.
- b.** Find the relative likelihood $P_{k=1}/P_{k=0}$ that a hydrogen molecule will be in its first excited vibrational state in thermal equilibrium at room temperature, $T = 293$ K.

6-4. (a) $\Delta E = E_1^{\text{vib}} - E_0^{\text{vib}} = h f = (6.626 \times 10^{-34}) (1.3 \times 10^{14}) \text{ J} = 8.6 \times 10^{-20} \text{ J} = 0.54 \text{ eV}$

(b) $P_1/P_0 = P_1/P_0 = e^{-(\Delta E)/k_B T} = e^{-0.54/(8.62 \times 10^{-5} \cdot 293)} = e^{-21.4} \approx 5 \times 10^{-10}$

6-5 Referring to problems 6-3 and 6-4 and Eq. (6-2), construct an energy level diagram for the H_2 molecule that shows the first vibrational and rotational states associated with the ground electronic state of the molecule. (*Hint:* The molecule can be vibrating and rotating at the same time.)

6-5. The first few states of the energy level diagram in the electronic ground state are shown below. The energy labels are of the form $E_{k,l}$. Associated with each vibrational state there are a number of rotational sublevels. An expanded view of each vibrational state with associated sublevels is shown at the right.



6-6 Consider an assembly of atoms that have two energy levels separated by an energy corresponding to a wavelength of $0.6328 \mu\text{m}$, as in the He-Ne laser. What is the ratio of the population densities of these two energy levels if the assembly of atoms is in thermal equilibrium at a temperature of $T = 300 \text{ K}$?

6-6. The energy separation of the states is $\Delta E = hc/\lambda = (1240 \text{ eV} \cdot \text{nm})/(632.8 \text{ nm}) = 01.96 \text{ eV}$.
Neglecting degeneracies: $N_2/N_1 = P_2/P_1 = e^{-(E_2-E_1)/k_B T} = e^{-1.96/(8.62 \times 10^{-5} \cdot 300)} \approx e^{-76}$.

6-7 The rate of decay of an assembly of atoms with population density N_2 at excited energy level E_2 when spontaneous emission is the only important process is

$$\left(\frac{dN_2}{dt} \right)_{\text{spont}} = -A_{21}N_2$$

Show that an initial population density N_{20} decreases to a value N_{20}/e in a time τ equal to $1/A_{21}$. That is, show that A_{21} is the inverse of the *lifetime* of the atomic level.

6-7. The differential equation can be integrated as shown below:

$$\begin{aligned} \frac{dN_2}{dt} &= -A_{21}N_2 \Rightarrow \frac{dN_2}{N_2} = -A_{21}dt \\ \int_{N_{20}}^{N_2(t)} \frac{dN_2}{N_2} &= -A_{21} \int_0^t dt' \Rightarrow \ln(N_2(t)/N_{20}) = -A_{21}(t-0) = -A_{21}t \\ \frac{N_2(t)}{N_{20}} &= e^{-A_{21}t} \Rightarrow N_2(t) = N_{20}e^{-A_{21}t} \end{aligned}$$

At time $t = 1/A_{21}$, $N_2(t) = N_{20}e^{-A_{21}(1/A_{21})} = N_{20}e^{-1} = N_{20}/e$

6-8 Derive the Wien displacement law from the Planck black-body spectral radiance formula.

6-8. The black-body spectral radiance is $M_\lambda = \frac{2 \pi h c^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_B T} - 1}$.

This function is maximized when its denominator is minimized:

$$\frac{d}{d\lambda} \left[\lambda^5 (e^{hc/\lambda k_B T} - 1) \right] = 0 \Rightarrow \lambda^5 \left[e^{hc/\lambda k_B T} \frac{hc}{k_B T} \left(-\frac{1}{\lambda^2} \right) + (e^{hc/\lambda k_B T} - 1) 5 \lambda^4 \right] = 0$$

Let $x = hc/(\lambda k_B T)$: $\lambda^4 [-x e^x + 5(e^x - 1)] = 0 \Rightarrow x e^x = 5 e^x - 1$. This is a transcendental equation in x with solution $x = 4.999986$. Then,

$$x_{\max} = 4.999986 = \frac{hc}{\lambda_{\max} k_B T} \Rightarrow \lambda_{\max} T = \frac{hc}{4.999986 k_B} = 2.898 \times 10^{-3} \text{ m} \cdot \text{K} = 2898 \text{ } \mu\text{m} \cdot \text{K}$$

6-9 Derive the Stefan-Boltzmann law from the Planck blackbody spectral radiance formula. (*Hint:* Use a substitution of $x = hc/\lambda k_B T$ to facilitate the integration.)

6-9. Let $x = hc/(\lambda k_B T)$. Then $dx = (hc/k_B T)(-d\lambda/\lambda^2)$ so that $d\lambda = -\lambda^2(k_B T/hc)dx$. Then,

$$M = \int_0^\infty M_\lambda d\lambda = \int_0^\infty \frac{2\pi h c^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_B T} - 1} d\lambda = \frac{2\pi (k_B T)^4}{h^3 c^2} \int_0^\infty \frac{x^3}{e^x - 1} = \frac{2\pi (k_B T)^4}{h^3 c^2} \frac{\pi^4}{15} = \left(\frac{2 k_B^4 \pi^5}{15 h^3 c^2} \right) T^4$$

$$M = \left(\frac{2 k_B^4 \pi^5}{15 h^3 c^2} \right) T^4 \equiv \sigma T^4 = (5.668 \times 10^{-8}) \text{ W/m}^2\text{-K}^4$$

- 6-10 a.** At what wavelength does a blackbody at 6000 K radiate the most per unit wavelength?
- b.** If the blackbody is a 1-mm diameter hole in a cavity radiator at this temperature, find the power radiated through the hole in the narrow wavelength region 5500–5510 Å.

6-10. As in the solution to problem (6-9),

$$M_\lambda = \frac{2\pi c^2}{\lambda^5} \frac{1}{e^x - 1}, \text{ with } x = \frac{hc}{\lambda k_B T}$$

Then,

$$M = \int_0^\infty M_\lambda d\lambda$$

The change of variable is accomplished by noting that for $\lambda = 0, x = \infty$ and for $\lambda = \infty, x = 0$. Further,

$$dx = \frac{hc}{k_B T} \left(-\frac{d\lambda}{\lambda^2} \right) \Rightarrow d\lambda = -\frac{\lambda^2 k_B T}{hc} dx$$

So,

$$M = 2\pi h c^2 \int_\infty^0 \frac{1}{\lambda^5} \frac{1}{e^x - 1} \frac{-\lambda^2 k_B T}{hc} dx = 2\pi c k_B T \int_0^\infty \frac{1}{\lambda^3} \frac{dx}{e^x - 1} = 2\pi c k_B T \int_0^\infty \left(\frac{x k_B T}{hc} \right)^3 \frac{dx}{e^x - 1}$$

$$M = \frac{2\pi (k_B T)^4}{h^3 c^2} \int_0^\infty \frac{x^3}{e^x - 1} dx$$

The definite integral has the value

$$\int_0^\infty \frac{x^3}{e^x - 1} dx = \frac{\pi^4}{15}$$

so that

$$M = \left(\frac{2k_B^4 \pi^5}{15 h^3 c^2} \right) T^4 \equiv \sigma T^4, \sigma = 5.668 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \text{K}^4}$$

6-11 At a given temperature, $\lambda_{\max} = 550 \text{ nm}$ for a blackbody cavity. The cavity temperature is then increased until its total radiant exitance is doubled. What is the new temperature and the new $\lambda_{\max}$?

6-11. The original temperature is determined from the Wien displacement law

$$\lambda_{\max} T_1 = 2898 \text{ } \mu\text{m} \cdot \text{K} \Rightarrow T_1 = \frac{2898 \text{ } \mu\text{m} \cdot \text{K}}{\lambda_{\max}} = \frac{2898 \text{ } \mu\text{m} \cdot \text{K}}{0.55 \text{ } \mu\text{m}} = 5269 \text{ K}$$

For the total radiant exitance to double,

$$M_2 = 2 M_1 \Rightarrow \sigma(T_2)^4 = 2\sigma T_1^4 \Rightarrow T_2 = 2^{1/4} T_1 = 6266 \text{ K}$$

The new maximum wavelength is

$$\lambda_{\max} = \frac{2898 \text{ } \mu\text{m} \cdot \text{K}}{T_2} = \frac{2898 \text{ } \mu\text{m} \cdot \text{K}}{6266 \text{ K}} = 0.462 \text{ } \mu\text{m} = 462 \text{ nm}$$

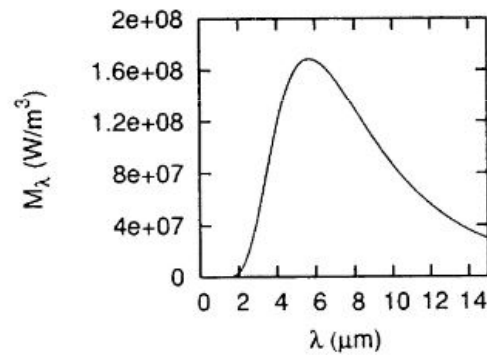
6-12 What must be the temperature of a graybody with emissivity of 0.45 if it is to have the same total radiant exitance as a blackbody at 5000 K?

6-12. The gray body at T_2 must match the black body at the lower temperature, T_1 . That is,

$$(M_{GB})_{T_2} = (M_{BB})_{T_1} \Rightarrow (\epsilon M_{BB})_{T_2} = (M_{BB})_{T_1}$$
$$\epsilon \sigma T_2^4 = \sigma T_1^4 \Rightarrow T_2^4 = T_1^4 / \epsilon = (5000 \text{ K})^4 / 0.45 \Rightarrow T_2 = 6105 \text{ K}$$

6-13 Plot the spectral exitance M_λ for a graybody of emissivity 0.4 in thermal equilibrium at 451°F, the temperature at which paper burns.

6-13. On the absolute temperature scale, 451 °F = 506 K. Then $M_\lambda = \varepsilon \frac{2\pi h c^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_B T} - 1}$. The plot is as shown.



6-14 Why should one expect lasing at ultraviolet wavelengths to be more difficult to attain than lasing at infrared wavelengths? Develop your answer based on the ratio A_{21}/B_{21} and the meaning of the A_{21} and B_{21} coefficients.

6-14. Equation (6-12) indicates that A_{21}/B_{21} is inversely proportional to the cube of the wavelength. This ratio is the ratio of spontaneous emission to stimulated emission. Stimulated emission contributes to the laser gain while spontaneous emission (essentially) does not. Thus, all else being equal, the gain on a transition resonant with short wavelength radiation is less than that of a transition resonant with long wavelength radiation.

6-15 The gain bandwidth of the lasing transition (that is, the width of the atomic lineshape $g(\nu)$ associated with the transition) in a Nd:YAG gain medium is about $\Delta\nu = 1.2 \times 10^{11}$ Hz. Express this bandwidth as a wavelength range $\Delta\lambda$. Use Table 6-1 to find the center wavelength of the Nd:YAG lasing transition.

6-15. The frequency bandwidth $\Delta\nu = \nu_2 - \nu_1$ is related to the wavelength range $\Delta\lambda = \lambda_2 - \lambda_1$ by

$$\Delta\nu = \frac{c}{\lambda_2} - \frac{c}{\lambda_1} = c \frac{\lambda_1 - \lambda_2}{\lambda_2 \lambda_1} \approx -c \frac{\Delta\lambda}{\lambda^2} \Rightarrow |\Delta\lambda| = \frac{\lambda^2 |\Delta\nu|}{c}$$

From Table 6-1, $\lambda = 1.06 \times 10^{-6}$ m, so $|\Delta\lambda| = \frac{(1.06 \times 10^{-6})^2 (1.2 \cdot 10^{11})}{(3 \cdot 10^8)} \text{ m} = 4.5 \times 10^{-10} \text{ m} = 0.45 \text{ nm}.$

6-16 The output of an argon ion laser can consist of a number of modes of frequency that match the cavity resonance condition and are within the gain bandwidth of the lasing transition. The gain bandwidth of the lasing transition is roughly

equal to the width of the atomic lineshape function $g(\nu)$ associated with the lasing transition. Take the gain bandwidth of an argon ion laser to be 2 GHz and the linewidth of an individual mode from the argon ion laser to be 100 kHz. The coherence time of a light beam is roughly equal to the reciprocal of the spread of frequencies present in light. Find the coherence time and the coherence length of the argon ion laser if

- a.** The laser output consists of a single mode.
- b.** The laser output consists of a number of modes with frequencies spread across the gain bandwidth of the lasing transition.

6-16. (a) $t_c \approx 1/\Delta\nu = 1/(10^5 \text{ Hz}) = 10^{-5} \text{ s}$ $L_c = ct_c = (3 \cdot 10^8 \text{ m/s}) (10^{-5} \text{ s}) = 3000 \text{ m}$

(b) $t_c \approx 1/\Delta\nu = 1/(2 \cdot 10^9 \text{ Hz}) = 5 \cdot 10^{-10} \text{ s}$ $L_c = ct_c = (3 \cdot 10^8 \text{ m/s}) (10^{-5} \text{ s}) = 0.15 \text{ m} = 15 \text{ cm}$

6-17 Find the number of standing wave cavity modes within the gain bandwidth of the argon ion laser of problem 6-16 if the laser system uses a resonator with flat mirrors separated by a distance $d = 0.5$ m.

6-17. The separation between cavity modes is $\Delta\nu = \nu_{m+1} - \nu_m = (m+1)c/2d - mc/2d = c/2d$. That is,

$$\Delta\nu = (3 \times 10^8 \text{ m/s}) / (2 \cdot 0.5 \text{ m}) = 0.3 \text{ GHz}$$

Then the number of lasing modes would be the ratio of the frequency range for lasing to the separation of cavity modes:

$$\# \text{ of lasing modes} = \frac{2 \text{ GHz}}{0.3 \text{ GHz}} = 6.67 \Rightarrow 6$$

6-18 A He-Ne laser has a beam waist (diameter) equal to about 1 mm.

- a.** What is its beam-spread angle in the far field?
- b.** Estimate the diameter of this beam after it has propagated over a distance of 1 km.

6-18. (a) The half-angle beam spread is

$$\theta = \frac{\lambda}{\pi w_0} = \frac{6.33 \cdot 10^{-7} \text{ m}}{\pi \cdot 5 \cdot 10^{-4} \text{ m}} = 4.0 \times 10^{-4} \text{ rad} = 0.023^\circ$$

(b) The diameter is about

$$D = 2 d \theta = 2 (1000 \text{ m}) (4.0 \times 10^{-4}) = 0.8 \text{ m} = 80 \text{ cm}$$

6-19 To what diameter spot should a He-Ne laser of power 10 mW be focused if the irradiance in the spot is to be the same as the sun's irradiance at the surface of the earth? (The irradiance of the sun at the earth's surface is about 1000 W/m^2 .)

6-19. Assuming that that irradiance is constant over the spot:

$$I_{\text{laser}} = \frac{P_{\text{laser}}}{A_{\text{laser}}} = \frac{P_{\text{laser}}}{\pi D^2/4} = I_{\text{sun}}$$
$$D = \sqrt{\frac{4 P_{\text{laser}}}{\pi I_{\text{sun}}}} = \sqrt{\frac{4 \cdot 0.01}{\pi 1000}} \text{ m} = 0.0036 \text{ m} = 3.6 \text{ mm}$$

6-20 For a Nd:YAG laser, there are four pump levels located at 1.53 eV, 1.653 eV, 2.119 eV, and 2.361 eV above the ground state energy level.

- a. What is the wavelength associated with the photon energy required to populate each of the pump levels?
- b. Knowing that a Nd:YAG laser emits photons of wavelength $1.064 \mu\text{m}$, determine the *quantum efficiency* associated with each of the four pump levels. (The quantum efficiency is the ratio of the energy of a single pump event to that of an output photon.)

6-20. (a) The energy difference ΔE between the levels must have near the same energy as the pump photons. That is,

$$\Delta E = h\nu = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ eV} \cdot \text{nm}}{\Delta E}$$

Then, $\lambda_{1.53} = (1240/1.53) \text{ nm} = 810 \text{ nm}$. Similarly, $\lambda_{1.653} = 750 \text{ nm}$, $\lambda_{2.119} = 585 \text{ nm}$, and $\lambda_{2.361} = 525 \text{ nm}$.

(b) The quantum efficiency η_Q is

$$\eta_Q = \frac{\text{Energy of an output photon}}{\text{Energy of a pump event}} = \frac{hc/\lambda_{\text{out}}}{hc/\lambda_p} = \frac{\lambda_p}{\lambda_{\text{out}}}$$

So using the results from part (a) for the various transitions:

$$\eta_{Q,1.53} = \frac{810}{1064} = 0.76, \quad \eta_{Q,1.653} = \frac{750}{1064} = 0.70, \quad \eta_{Q,2.119} = \frac{585}{1064} = 0.55, \quad \text{and} \quad \eta_{Q,2.361} = \frac{525}{1064} = 0.49.$$

6-21 To operate a Nd:YAG laser, 2500 W of “wall-plug” power are required for a power supply that drives the arc lamps. The arc lamps provide pump energy to create the population inversion. The overall laser system, from power in (to the power supply) to power out (laser output beam), is characterized by the following component efficiencies:

80%—power supply operation

30%—arc lamps for pump light energy

70%—optical reflectors for concentrating pump light on laser rod

15%—for spectral match of pump light to Nd:YAG pump levels

50%—due to internal cavity/rod losses

- a.** Taking the efficiencies into account sequentially as they “occur,” how much of the initial 2500 W is available for power in the output beam?
- b.** What is the overall operational efficiency (wall-plug efficiency) for this laser?

6-21. (a) The laser output beam would have a power of $P_{\text{output}} = 0.8 \times 0.3 \times 0.7 \times 0.15 \times 0.5 \times 2500 \text{ W} = 31.5 \text{ W}$.

(b) The overall efficiency is $\eta = \frac{P_{\text{output}}}{P_{\text{pump}}} = \frac{31.5}{2500} = 0.0126$.

6-22 Table 6-1 indicates that diode lasers have a large divergence angle. Why is this reasonable?

6-22. The diode laser beam is generated in a very small region. The radius w_0 of the beam waist of the diode laser beam must therefore be very small (compared to other types of lasers). As indicated by Equation (6-16), the divergence angle is inversely proportional to w_0 and so will be larger for diode lasers than for other types of lasers.

6-23 As the irradiance within a laser cavity increases in the build up to steady state, does the population inversion in the gain medium increase or decrease? Explain.

6-23. As the irradiance in the laser cavity increases, the population inversion decreases. Given that there is a positive population inversion, the population densities of the upper and lower lasing levels satisfy the inequality $N_{\text{upper}} > N_{\text{lower}}$. The magnitude of the net rate of depletion of the population inversion due to stimulated processes is proportional to $I (N_{\text{upper}} - N_{\text{lower}})$. Thus, increased irradiance causes a larger rate of depletion of the population inversion, leading to a lessened inversion.

6-24 Why is a Nd:YAG laser system that uses a diode laser as a pump more efficient than a Nd:YAG laser system that uses an arc lamp as a pump? See Table 6-1.

6-24. The diode laser frequency can be tuned to match one of the pump transitions (see problem 6-20) of the Nd:YAG gain medium. Thus, nearly all of the diode laser pump energy can be used to pump the laser. Much of the arc lamp energy—see Figures (6-8) and (6-9)—is not resonant with a pump transition and so is wasted. This wasted pump energy lowers the overall efficiency of the laser system.