



狹義相對論簡介



狹義相對論的基本假設（原理）

1. 狹義相對性原理 (principle of relativity) :
物理定律在所有慣性座標系中都相同。
2. 光速不變原理（許多實驗的結果） :
對於所有慣性座標系，光在真空 (free space)
中的速度皆相同 ($c \approx 3 \times 10^8 \text{ m/s}$)，與光源的速度無關。

沒有哪一個慣性座標系是特別的！

看似簡單的假設，卻可得出許多重要的結論！

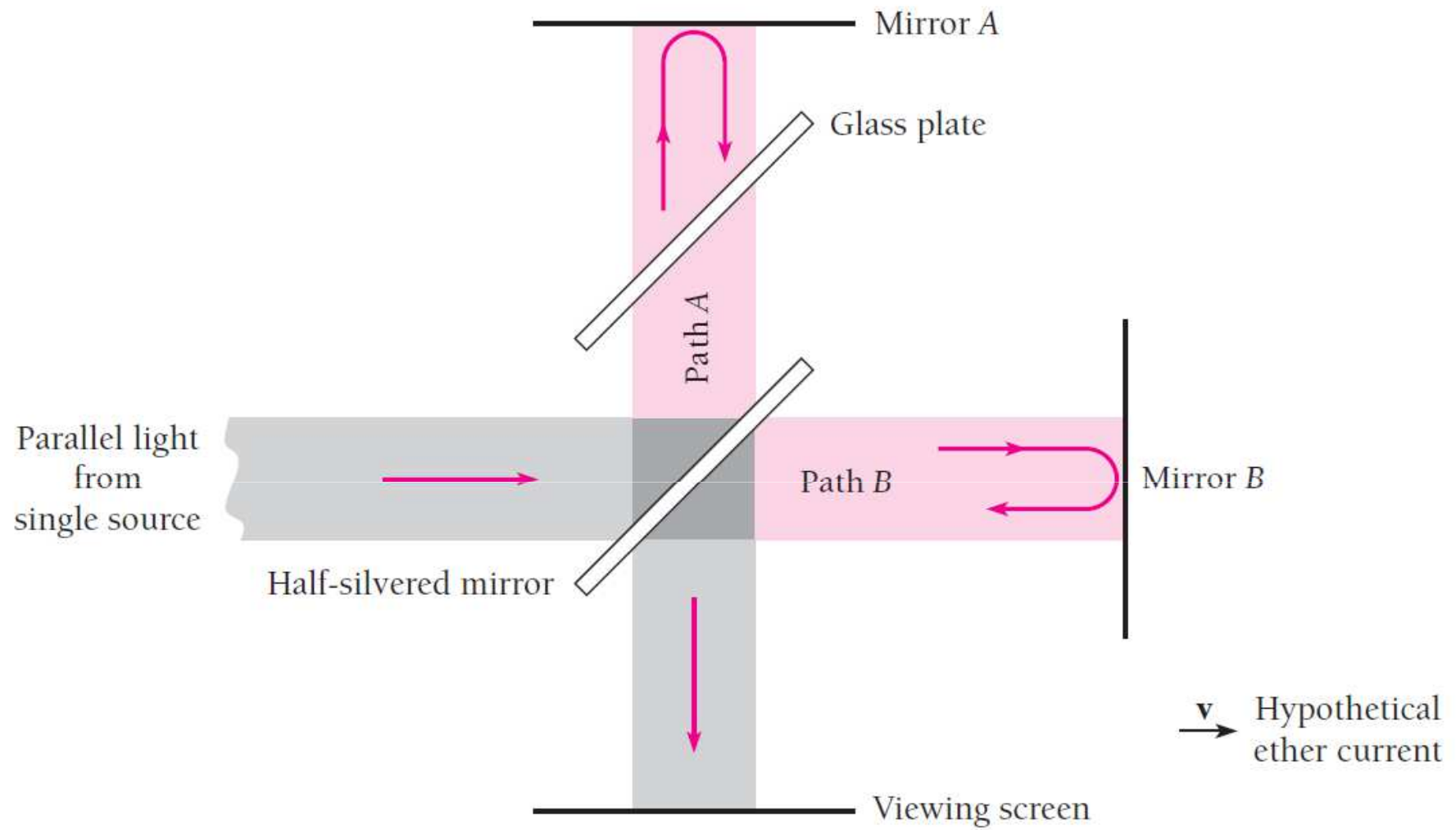


Figure 1.2 The Michelson-Morley experiment.

狹義相對論的重要結論

1. 時間膨脹 (time dilation) : $t = \gamma t_0$
2. 長度收縮 (length contraction) : $L = L_0 / \gamma$
3. 動量、能量修正：

$$\mathbf{p} = \gamma m \mathbf{v}$$

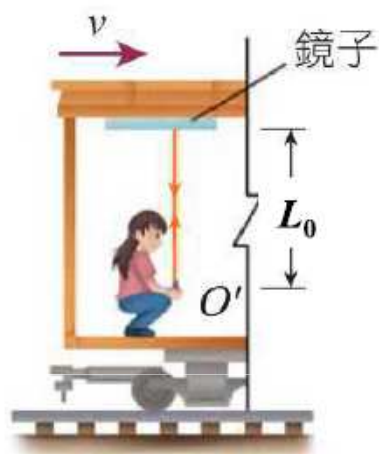
$$\text{KE} = \gamma mc^2 - mc^2 \quad \text{或} \quad E = \text{KE} + mc^2 = \gamma mc^2$$

$$\text{相對論修正因子：} \gamma \equiv \frac{1}{\sqrt{1 - v^2/c^2}} > 1$$

(當 $v/c \ll 1$ ，則 $\gamma \approx 1$)

時間膨脹

運動中的時鐘，走的比靜止時要慢

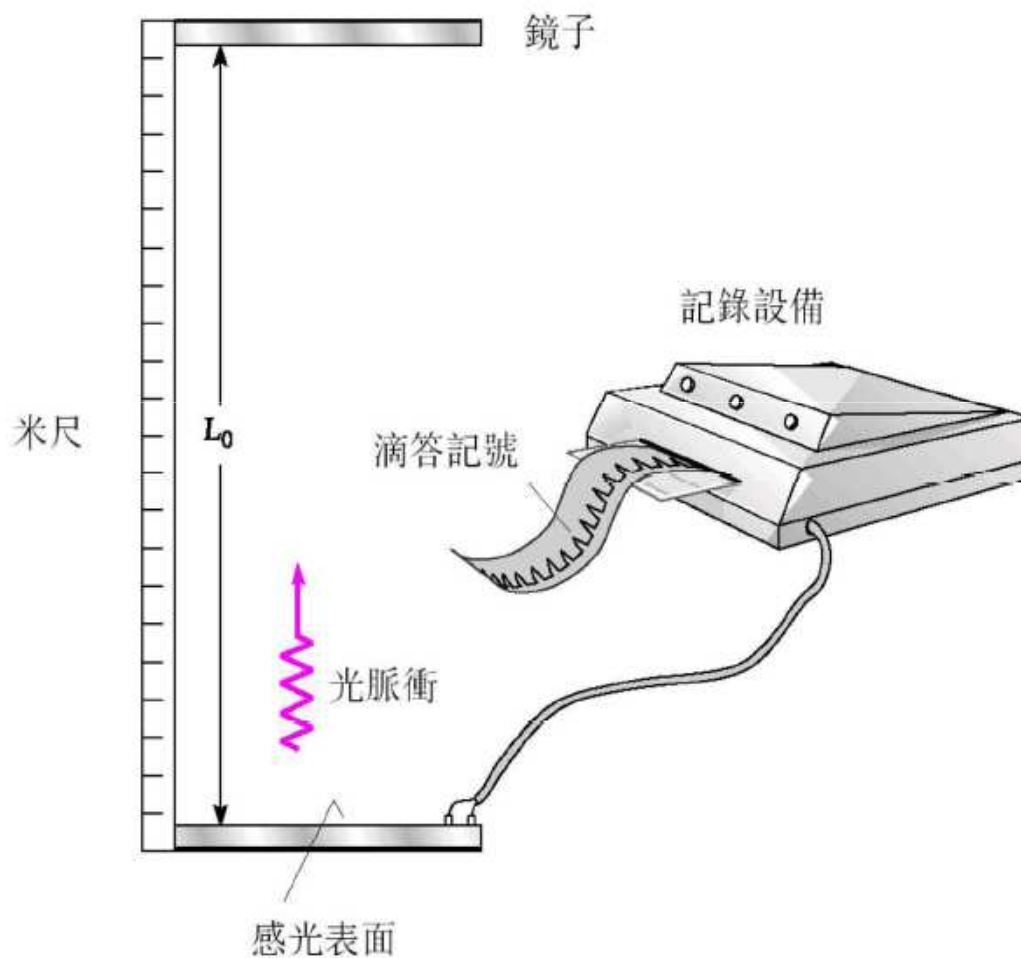


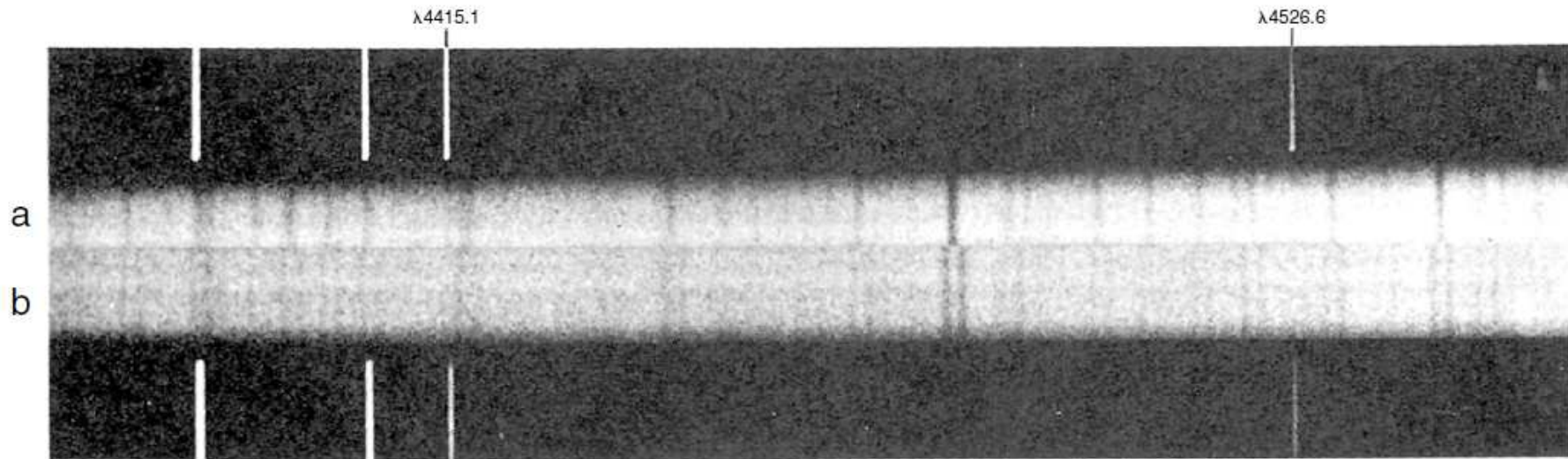
(a)

(a)小莉 O' 的時間間隔

$$t_0 = 2L_0/c$$

t_0 ：原時 (proper time)

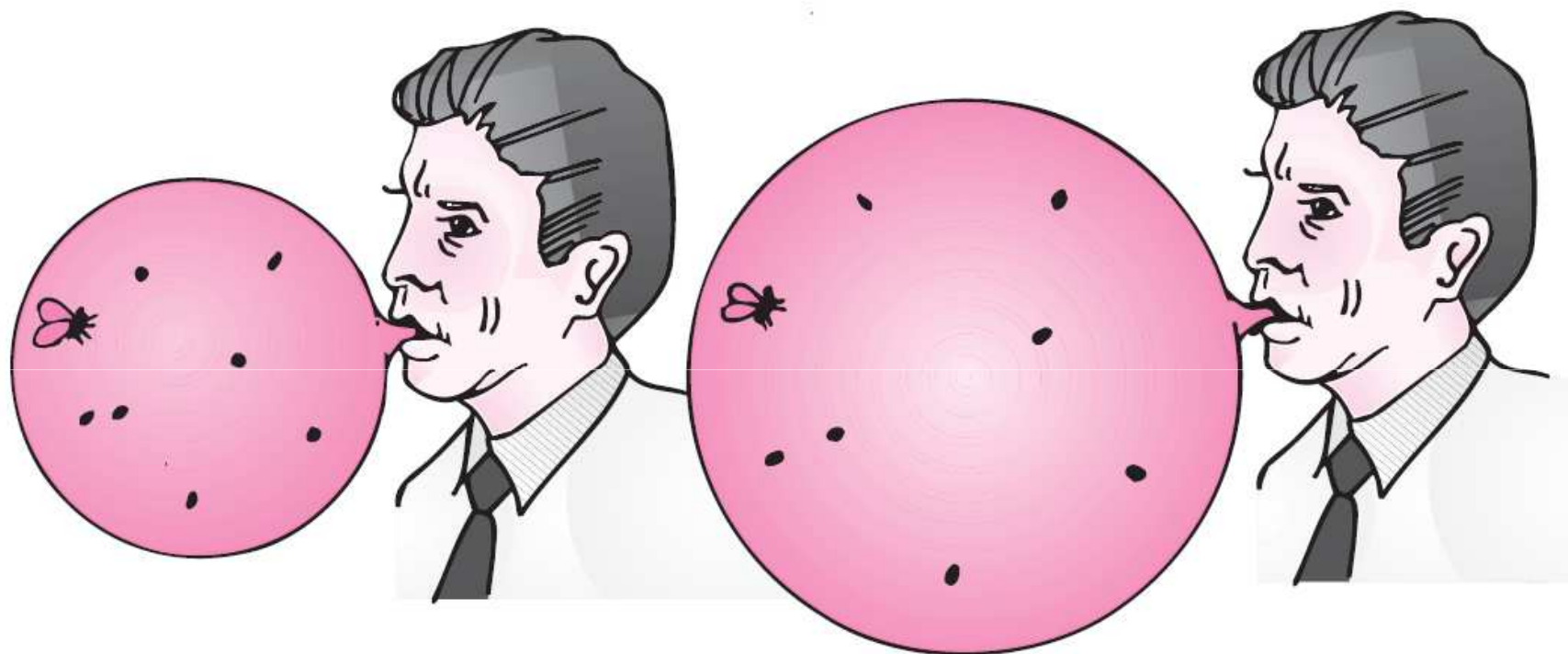




Longitudinal
doppler effect
in light

$$\nu = \nu_0 \sqrt{\frac{1 + v/c}{1 - v/c}} \quad (1.8)$$

by adopting the convention that v is $+$ for source and observer approaching each other and $-$ for source and observer receding from each other.



Example 1.1

A spacecraft is moving relative to the earth. An observer on the earth finds that, between 1 P.M. and 2 P.M. according to her clock, 3601 s elapse on the spacecraft's clock. What is the spacecraft's speed relative to the earth?

Solution

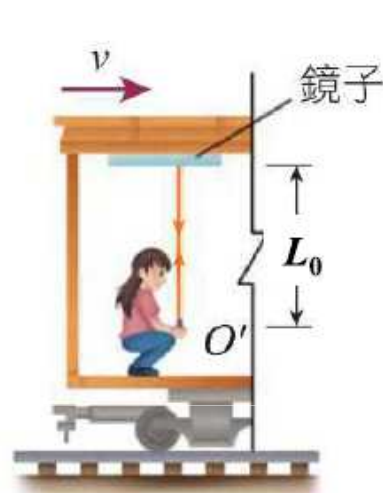
Here $t_0 = 3600$ s is the proper time interval on the earth and $t = 3601$ s is the time interval in the moving frame as measured from the earth. We proceed as follows:

$$\begin{aligned} t &= \frac{t_0}{\sqrt{1 - v^2/c^2}} \\ 1 - \frac{v^2}{c^2} &= \left(\frac{t_0}{t} \right)^2 \\ v &= c \sqrt{1 - \left(\frac{t_0}{t} \right)^2} = (2.998 \times 10^8 \text{ m/s}) \sqrt{1 - \left(\frac{3600 \text{ s}}{3601 \text{ s}} \right)^2} \\ &= 7.1 \times 10^6 \text{ m/s} \end{aligned}$$

Today's spacecraft are much slower than this. For instance, the highest speed of the Apollo 11 spacecraft that went to the moon was only 10,840 m/s, and its clocks differed from those on the earth by less than one part in 10^9 . Most of the experiments that have confirmed time dilation made use of unstable nuclei and elementary particles which readily attain speeds not far from that of light.

時間膨脹

運動中的時鐘，走的比靜止時要慢

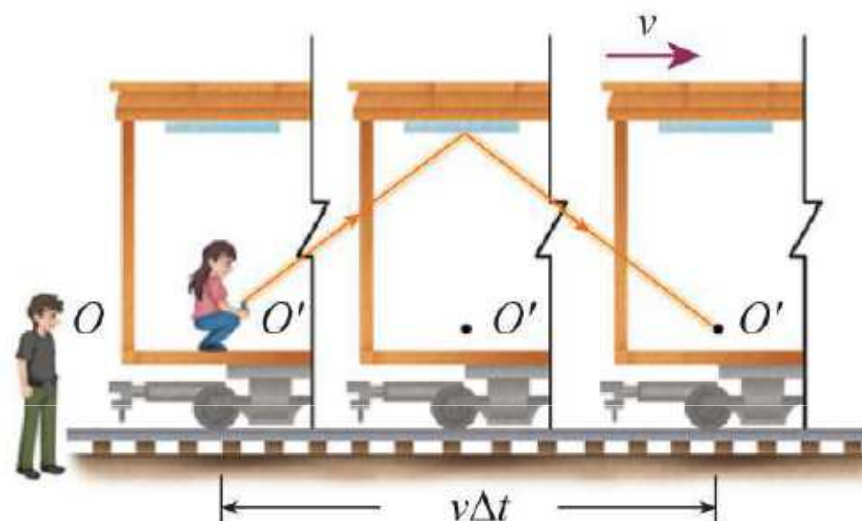


(a)

(a) 小莉 O' 的時間間隔

$$t_0 = 2L_0/c$$

t_0 : 原時 (proper time)



(b)

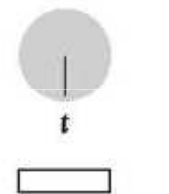
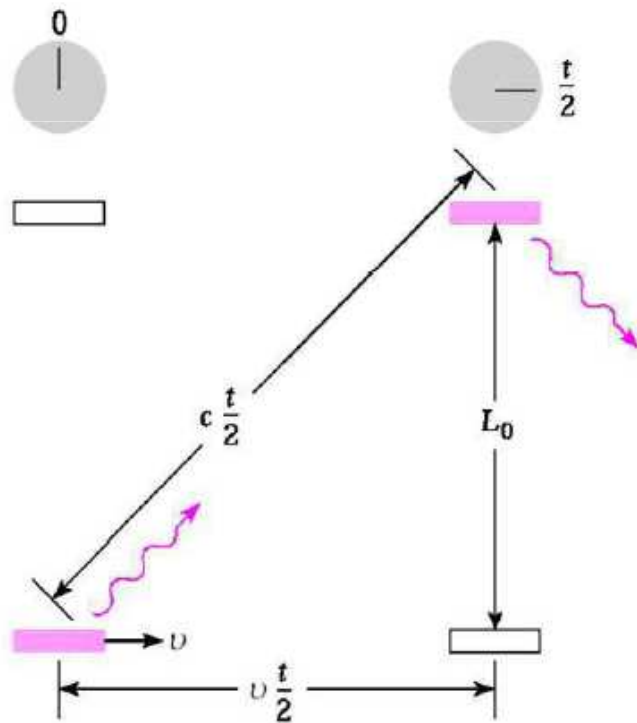
(b) 在行進列車上所發生的事件對於在月台的小山 O 來說，經過的時間為

$$t = \frac{2L_0/c}{\sqrt{1-v^2/c^2}} = \gamma t_0$$

光速 c 對於二位觀察者均相同。

然而，對觀察者 O 而言，光自 O' 發出而抵達鏡子，這段距離比較 O' 所見來得遠。

因此，由 O 所測得二事件所經歷時間 t 必較由 O' 所測得者 t_0 為長。
此一現象即為時間膨脹。



$$\left(\frac{ct}{2}\right)^2 = L_0^2 + \left(\frac{vt}{2}\right)^2$$

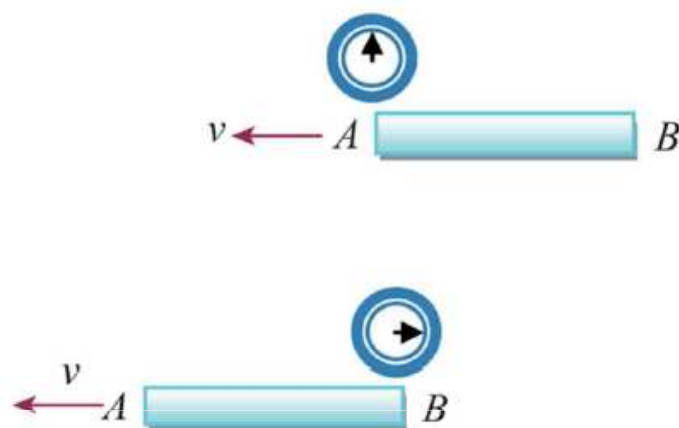
$$\frac{t^2}{4}(c^2 - v^2) = L_0^2$$

$$t^2 = \frac{4L_0^2}{c^2 - v^2} = \frac{(2L_0)^2}{c^2(1 - v^2/c^2)}$$

$$t = \frac{2L_0/c}{\sqrt{1 - v^2/c^2}} \quad (1.2)$$

$1/\gamma$

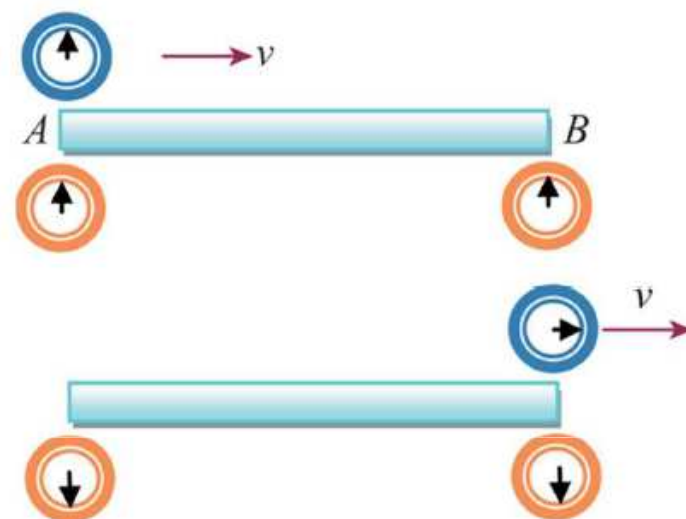
長度收縮 運動中的尺，沿著運動方向的長度會縮短



對於持有藍色時鐘的小莉 O' 而言，棍子以速度 v 向左運動。當棍子的 A、B 兩端分別經過小莉，測得的時間間隔為 t_0 ，則小莉測得的棍長為

$$L = v t_0。$$

$$L = v t / \gamma = L_0 / \gamma$$



相對於棍子靜止的小山 O 來說，棍長為 L_0 ，而小莉 O' 以速度 v 向右運動。經過的時間為 $t = \gamma t_0$ ，所以 $L_0 = v t$ 。

L_0 ：原長 (proper length)

the muon lifetime of $t_0 = 2.2 \mu\text{s}$

$$t = \frac{t_0}{\sqrt{1 - v^2/c^2}} = \frac{2.2 \times 10^{-6} \text{ s}}{\sqrt{1 - (0.998c)^2/c^2}}$$

$$= 34.8 \times 10^{-6} \text{ s} = 34.8 \mu\text{s}$$

$$vt = (2.994 \times 10^8 \text{ m/s})(34.8 \times 10^{-6} \text{ s})$$

$$= 1.04 \times 10^4 \text{ m} = 10.4 \text{ km}$$

Muon are actually created at altitudes of 6 km or more.

$$h = h_0 \sqrt{1 - v^2/c^2}$$

$$h = (10.4 \text{ km}) \sqrt{1 - (0.998c)^2/c^2} = 0.66 \text{ km}$$

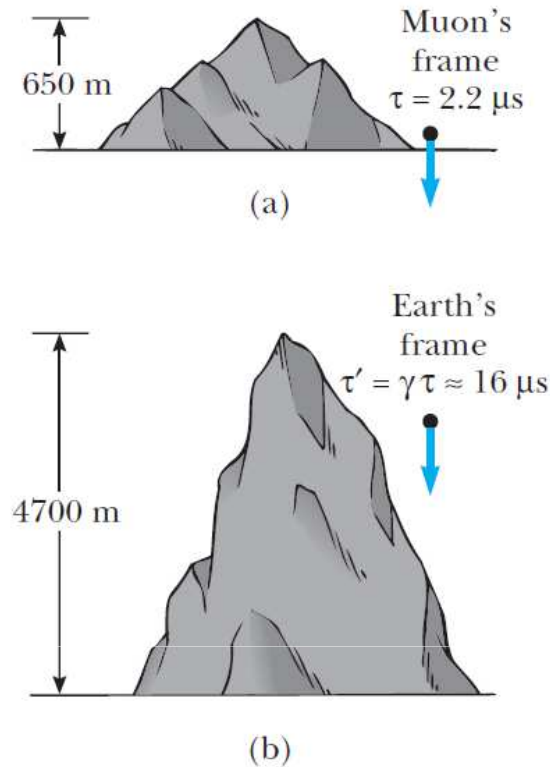
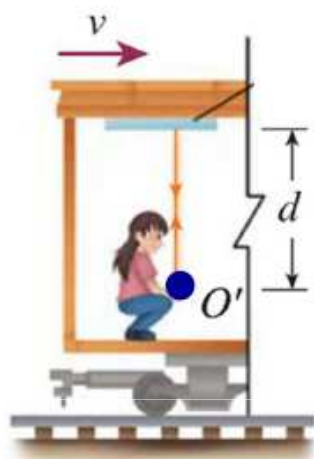


Figure 1.11 (a) Muons traveling with a speed of $0.99c$ travel only about 650 m as measured in the muons' reference frame, where their lifetime is about $2.2 \mu\text{s}$. (b) The muons travel about 4700 m as measured by an observer on Earth. Because of time dilation, the muons' lifetime is longer as measured by the Earth observer.

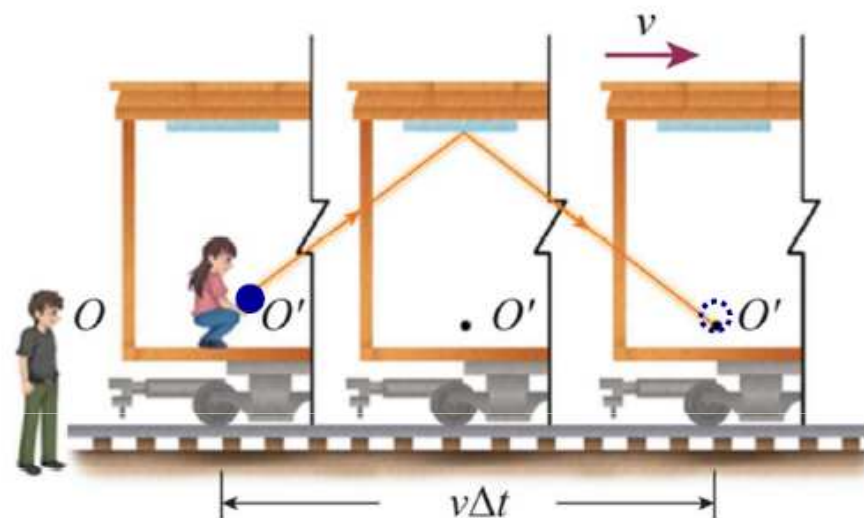
相對論的速度轉換 (垂直於相對運動的方向)



(a)

對於小莉 O' 而言，球來回一次的時間為 t_0 ，所以

$$v'_y = d/(t_0/2)$$



(b)

對於在月台的小山 O 來說，經過的時間為 $t = \gamma t_0$ ，所以

$$v_y = d/(t/2)$$

$$v_y = 2d/(\gamma t_0) = v'_y / \gamma$$

相對論的線動量與第二運動定律

若要求動量守恒，對於一個以速度 v 移動的物體，其質量應為

$$m(v) = \frac{m}{\sqrt{1 - v^2 / c^2}} = \gamma m \quad \text{運動中的物體質量比較大！}$$

m 為相對於觀察者而言靜止物體的原質量 (proper mass)；或稱為靜止質量 (rest mass)

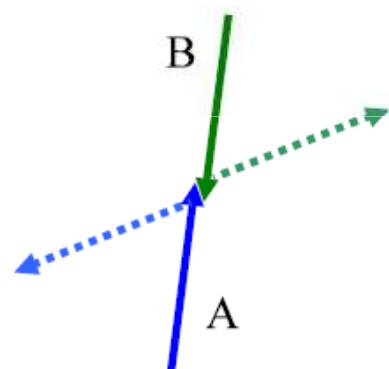
$$\Rightarrow \text{相對論的線動量} \quad \mathbf{p} = \frac{m\mathbf{v}}{\sqrt{1 - v^2 / c^2}} = \gamma m\mathbf{v}$$

$$\text{相對論第二運動定律} \quad \mathbf{F} = \frac{d\mathbf{p}}{dt} = \frac{d}{dt}(\gamma m\mathbf{v})$$

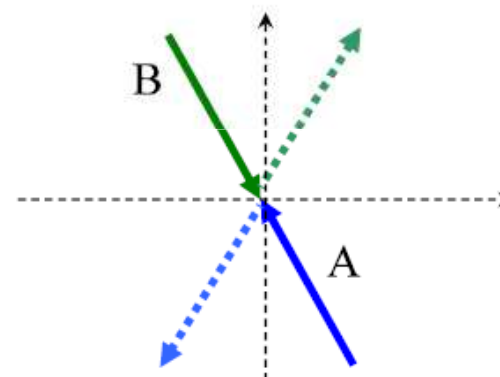
考慮兩相同粒子 AB 以相同速率相互接近，作彈性碰撞。

碰撞前，兩者總動量為零。

碰撞後，由動量守恆，兩者以相同速率分開。



選擇座標系統

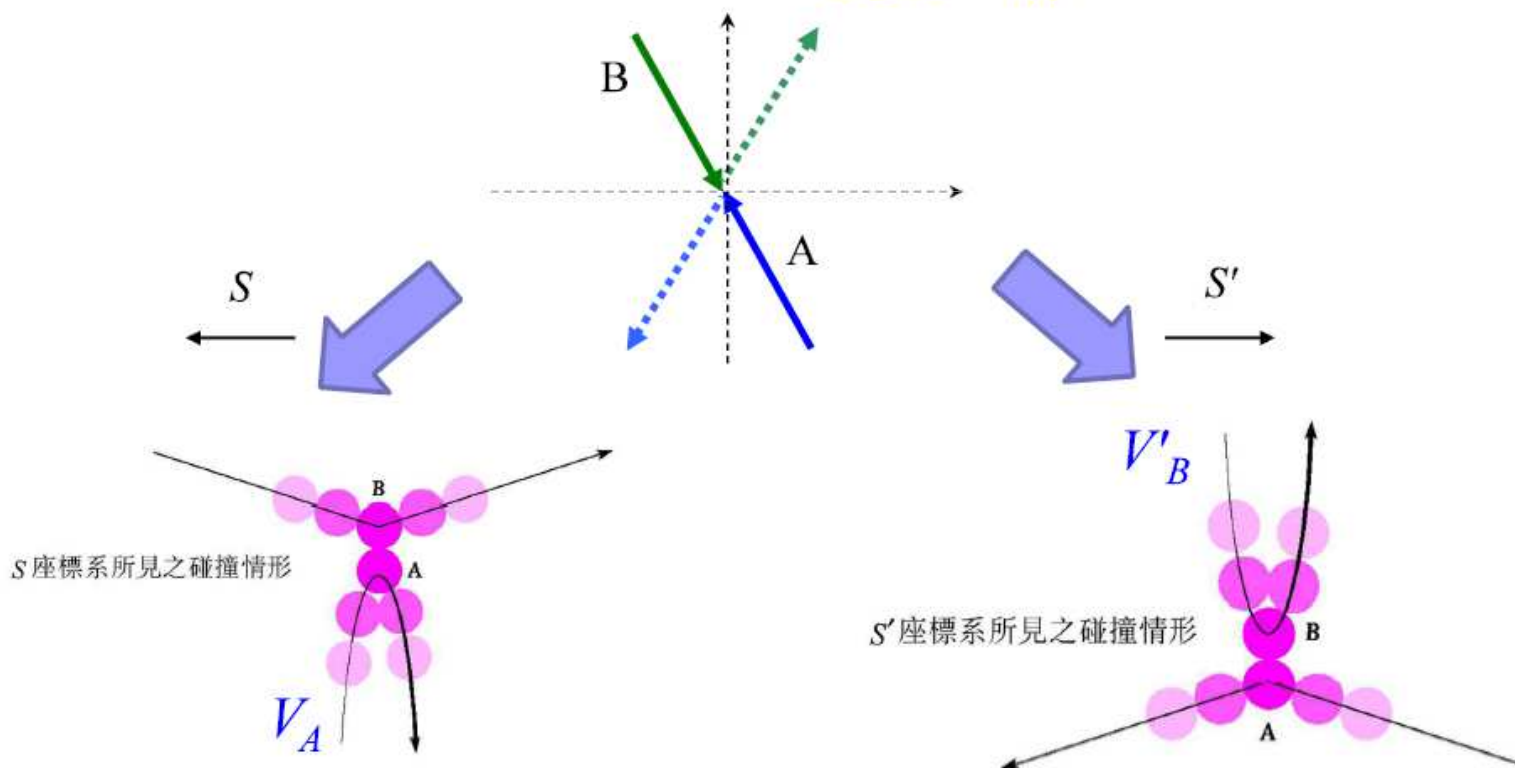


y 方向的總動量為零

選擇某一座標系 S ，使得在碰撞前， A 以速率 V_A 被拋向 $+y$ 方向；
選擇另一座標系 S' ，與 S 相對速度為 v ，且在碰撞前， B 以速率 V'_B 被拋向 $-y'$ 方向。所以有

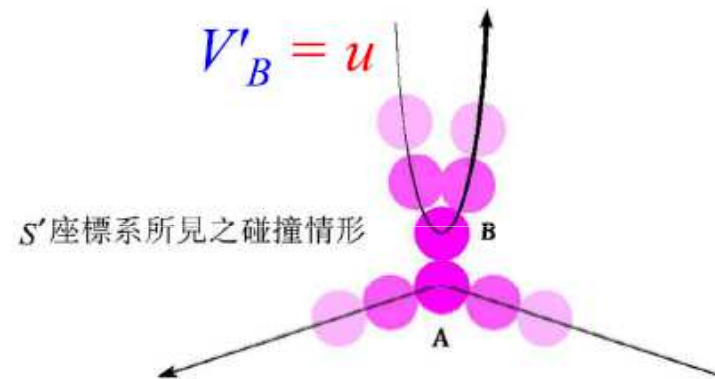
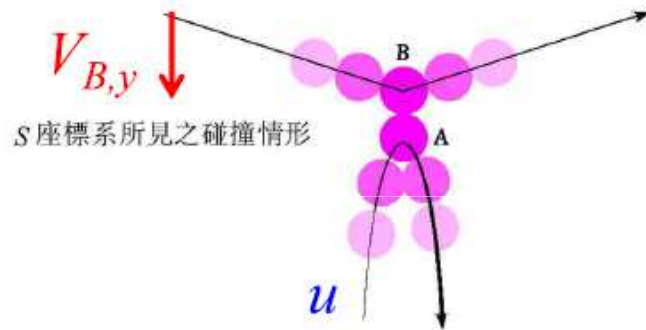
$$V_A = V'_B \equiv u$$

因此由 S 觀察 A 和由 S' 觀察 B 的性質完全一樣。



由相對論的速度轉換公式（垂直於相對運動的方向）可得，
在 S 座標系中， B 的 y 方向速度為

$$V_{B,y} = -\frac{V'_B}{\gamma} = -\sqrt{1-v^2/c^2} u \quad \left(\text{座標系 } S \text{ 與 } S' \text{ 的相對速度為 } v \right)$$



⇒ 在 S 座標系中觀察兩者 y 方向動量為

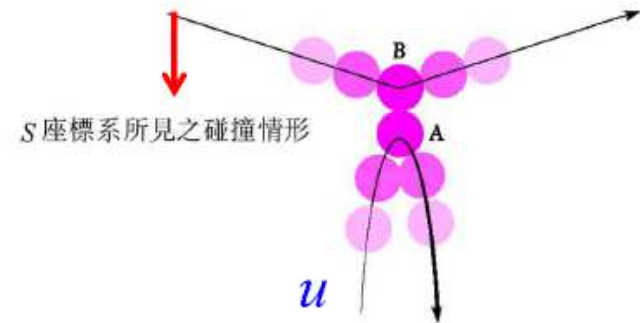
$$p_{A,y} = m(u)u$$

$$p_{B,y} = -m(V_B)\sqrt{1-v^2/c^2} u$$

在 S 座標系中觀察兩者 y 方向動量為

$$p_{A,y} = m(u)u$$

$$p_{B,y} = -m(V_B)\sqrt{1-v^2/c^2}u$$



為使 y 方向的總動量為零，則速率不相同的物體的質量比應為

$$\frac{m(V_B)}{m(u)} = \frac{1}{\sqrt{1-v^2/c^2}} \neq 1$$

當 $u \rightarrow 0$ ，則 $V_B \rightarrow v$ ，且 $m(V_B) \rightarrow m(v)$ ， $m(u) \rightarrow m(0) \equiv m$ ，則上式可簡化為

$$m(v) = \frac{m}{\sqrt{1-v^2/c^2}} = \gamma m$$

$$\Rightarrow \text{相對論的線動量} \quad \mathbf{p} = \frac{m\mathbf{v}}{\sqrt{1-v^2/c^2}} = \gamma m\mathbf{v}$$

相對論的動能與靜止能量

∴ 相對論第二運動定律 $\mathbf{F} = \frac{d\mathbf{p}}{dt} = \frac{d}{dt}(\gamma m\mathbf{v})$

∴ 動能 $\text{KE} = \int_0^s F ds = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2$

總能量 $E = \text{KE} + mc^2 = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - v^2 / c^2}}$

靜止能量 $E_0 = mc^2$

作用於質點上的力所做的功，等於該質點的動能變化

$$\text{KE} = \int_0^s F ds = \int_0^s \frac{d(\gamma mv)}{dt} ds = \int_0^v \frac{ds}{dt} d(\gamma mv)$$

$$= \int_0^v v d\left(\frac{mv}{\sqrt{1-v^2/c^2}}\right)$$

$$= \frac{mv^2}{\sqrt{1-v^2/c^2}} - m \int_0^v v \frac{dv}{\sqrt{1-v^2/c^2}}$$

$$\int x dy = xy - \int y dx$$

$$= \frac{mv^2}{\sqrt{1-v^2/c^2}} + m \left[c^2 \sqrt{1-v^2/c^2} \right]_0^v$$

$$= \frac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2 = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2$$

狹義相對論的重要公式總整理

線動量 $\mathbf{p} = \frac{m\mathbf{v}}{\sqrt{1-v^2/c^2}} = \gamma m\mathbf{v}$

動能 $\text{KE} = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2$

總能量 $E = \text{KE} + mc^2 = \gamma mc^2 = \frac{mc^2}{\sqrt{1-v^2/c^2}}$

⇒ 能量與動量之關係式 $E^2 - (pc)^2 = (mc^2)^2$

當質點靜止不動時 $p = 0$ ，此時該質點的 $E = mc^2$

對一個沒有靜質量的質點 ($m = 0$) 而言，它的能量 $E = pc$

Example 1.6

A stationary body explodes into two fragments each of mass 1.0 kg that move apart at speeds of $0.6c$ relative to the original body. Find the mass of the original body.

Solution

The rest energy of the original body must equal the sum of the total energies of the fragments. Hence

$$E_0 = mc^2 = \gamma m_1 c^2 + \gamma m_2 c^2 = \frac{m_1 c^2}{\sqrt{1 - v_1^2/c^2}} + \frac{m_2 c^2}{\sqrt{1 - v_2^2/c^2}}$$

and

$$m = \frac{E_0}{c^2} = \frac{(2)(1.0 \text{ kg})}{\sqrt{1 - (0.60)^2}} = 2.5 \text{ kg}$$

Example 1.7

Solar energy reaches the earth at the rate of about 1.4 kW per square meter of surface perpendicular to the direction of the sun (Fig. 1.15). By how much does the mass of the sun decrease per second owing to this energy loss? The mean radius of the earth's orbit is 1.5×10^{11} m.

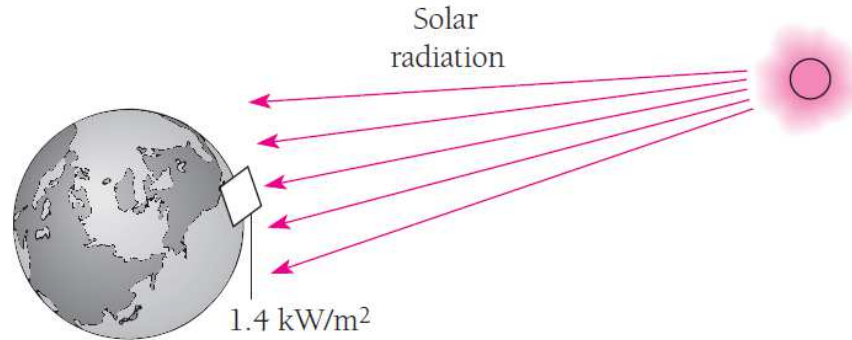


Figure 1.15

Solution

The surface area of a sphere of radius r is $A = 4\pi r^2$. The total power radiated by the sun, which is equal to the power received by a sphere whose radius is that of the earth's orbit, is therefore

$$P = \frac{P}{A} A = \frac{P}{A} (4\pi r^2) = (1.4 \times 10^3 \text{ W/m}^2)(4\pi)(1.5 \times 10^{11} \text{ m})^2 = 4.0 \times 10^{26} \text{ W}$$

Thus the sun loses $E_0 = 4.0 \times 10^{26}$ J of rest energy per second, which means that the sun's rest mass decreases by

$$m = \frac{E_0}{c^2} = \frac{4.0 \times 10^{26} \text{ J}}{(3.0 \times 10^8 \text{ m/s})^2} = 4.4 \times 10^9 \text{ kg}$$

per second. Since the sun's mass is 2.0×10^{30} kg, it is in no immediate danger of running out of matter. The chief energy-producing process in the sun and most other stars is the conversion of hydrogen to helium in its interior. The formation of each helium nucleus is accompanied by the release of 4.0×10^{-11} J of energy, so 10^{37} helium nuclei are produced in the sun per second.