

An unstable high-energy particle enters a detector and leaves a track of length 1.05 mm before it decays. Its speed relative to the detector was $0.992c$. What is its proper lifetime? That is, how long would the particle have lasted before decay had it been at rest with respect to the detector?

3. In the laboratory, it travels a distance $d = 0.00105 \text{ m} = vt$, where $v = 0.992c$ and t is the time measured on the laboratory clocks. We can use Eq. 37-7 to relate t to the proper lifetime of the particle t_0 :

$$t = \frac{t_0}{\sqrt{1 - (v/c)^2}} \Rightarrow t_0 = t \sqrt{1 - \left(\frac{v}{c}\right)^2} = \frac{d}{0.992c} \sqrt{1 - 0.992^2}$$

which yields $t_0 = 4.46 \times 10^{-13} \text{ s} = 0.446 \text{ ps}$.

(Come) back to the future. Suppose that a father is 20.00 y older than his daughter. He wants to travel outward from Earth for 2.000 y and then back to Earth for another 2.000 y (both intervals as he measures them) such that he is then 20.00 y *younger* than his daughter. What constant speed parameter β (relative to Earth) is required for the trip?

6. Due to the time-dilation effect, the time between initial and final ages for the daughter is longer than the four years experienced by her father:

$$t_{f \text{ daughter}} - t_{i \text{ daughter}} = \gamma(4.000 \text{ y})$$

where γ is Lorentz factor (Eq. 37-8). Letting T denote the age of the father, then the conditions of the problem require

$$T_i = t_{i \text{ daughter}} + 20.00 \text{ y}, \quad T_f = t_{f \text{ daughter}} - 20.00 \text{ y}.$$

Since $T_f - T_i = 4.000 \text{ y}$, then these three equations combine to give a single condition from which γ can be determined (and consequently v):

$$44 = 4\gamma \Rightarrow \gamma = 11 \Rightarrow \beta = \frac{2\sqrt{30}}{11} = 0.9959.$$

A spaceship of rest length 130 m races past a timing station at a speed of $0.740c$. (a) What is the length of the spaceship as measured by the timing station? (b) What time interval will the station clock record between the passage of the front and back ends of the ship?

11. (a) The rest length $L_0 = 130$ m of the spaceship and its length L as measured by the timing station are related by Eq. 37-13. Therefore,

$$L = L_0 \sqrt{1 - (v/c)^2} = (130 \text{ m}) \sqrt{1 - (0.740)^2} = 87.4 \text{ m}.$$

(b) The time interval for the passage of the spaceship is

$$\Delta t = \frac{L}{v} = \frac{87.4 \text{ m}}{(0.740)(3.00 \times 10^8 \text{ m/s})} = 3.94 \times 10^{-7} \text{ s}.$$

The center of our Milky Way galaxy is about 23 000 ly away. (a) To eight significant figures, at what constant speed parameter would you need to travel exactly 23 000 ly (measured in the Galaxy frame) in exactly 30 y (measured in your frame)? (b) Measured in your frame and in light-years, what length of the Galaxy would pass by you during the trip?

13. (a) Let $d = 23000 \text{ ly} = 23000 c \text{ y}$, which would give the distance in meters if we included a conversion factor for years $\rightarrow$ seconds. With $\Delta t_0 = 30 \text{ y}$ and $\Delta t = d/v$ (see Eq. 37-10), we wish to solve for v from Eq. 37-7. Our first step is as follows:

$$\Delta t = \frac{d}{v} = \frac{\Delta t_0}{\sqrt{1-\beta^2}} \Rightarrow \frac{23000 \text{ y}}{\beta} = \frac{30 \text{ y}}{\sqrt{1-\beta^2}},$$

at which point we can cancel the unit year and manipulate the equation to solve for the speed parameter β . This yields

$$\beta = \frac{1}{\sqrt{1+(30/23000)^2}} = 0.99999915.$$

(b) The Lorentz factor is $\gamma = 1/\sqrt{1-\beta^2} = 766.6680752$. Thus, the length of the galaxy measured in the traveler's frame is

$$L = \frac{L_0}{\gamma} = \frac{23000 \text{ ly}}{766.6680752} = 29.99999 \text{ ly} \approx 30 \text{ ly}.$$

•40 What is the minimum energy that is required to break a nucleus of ^{12}C (of mass 11.996 71 u) into three nuclei of ^4He (of mass 4.001 51 u each)?

40. From Eq. 28-37, we have

$$\begin{aligned} Q &= -\Delta Mc^2 = -[3(4.00151\text{u}) - 11.99671\text{u}]c^2 = -(0.00782\text{u})(931.5\text{MeV/u}) \\ &= -7.28\text{MeV}. \end{aligned}$$

Thus, it takes a minimum of 7.28 MeV supplied to the system to cause this reaction. We note that the masses given in this problem are strictly for the nuclei involved; they are not the “atomic” masses that are quoted in several of the other problems in this chapter.

•41 How much work must be done to increase the speed of an electron (a) from $0.18c$ to $0.19c$ and (b) from $0.98c$ to $0.99c$? Note that the speed increase is $0.01c$ in both cases.

41. (a) The work-kinetic energy theorem applies as well to relativistic physics as to Newtonian; the only difference is the specific formula for kinetic energy. Thus, we use $W = \Delta K$ where $K = m_e c^2 (\gamma - 1)$ (Eq. 37-52), and $m_e c^2 = 511 \text{ keV} = 0.511 \text{ MeV}$ (Table 37-3). Noting that

$$\Delta K = m_e c^2 (\gamma_f - \gamma_i),$$

we obtain

$$\begin{aligned} W = \Delta K &= m_e c^2 \left(\frac{1}{\sqrt{1 - \beta_f^2}} - \frac{1}{\sqrt{1 - \beta_i^2}} \right) = (511 \text{ keV}) \left(\frac{1}{\sqrt{1 - (0.19)^2}} - \frac{1}{\sqrt{1 - (0.18)^2}} \right) \\ &= 0.996 \text{ keV} \approx 1.0 \text{ keV}. \end{aligned}$$

(b) Similarly,

$$W = (511 \text{ keV}) \left(\frac{1}{\sqrt{1 - (0.99)^2}} - \frac{1}{\sqrt{1 - (0.98)^2}} \right) = 1055 \text{ keV} \approx 1.1 \text{ MeV}.$$

We see the dramatic increase in difficulty in trying to accelerate a particle when its initial speed is very close to the speed of light.

What must be the momentum of a particle with mass m so that the total energy of the particle is 3.00 times its rest energy?

45. We set Eq. 37-55 equal to $(3.00mc^2)^2$, as required by the problem, and solve for the speed. Thus,

$$(pc)^2 + (mc^2)^2 = 9.00(mc^2)^2$$

leads to $p = mc\sqrt{8} \approx 2.83mc$.

The mass of an electron is $9.10938188 \times 10^{-31}$ kg. To eight significant figures, find γ, β for the given electron kinetic energy: $K=1.000\ 000\ 0$ MeV. (02小題)

(c) For $K = 1.0000000$ MeV, we have

$$\gamma = \frac{K}{m_e c^2} + 1 = \frac{1.0000000 \text{ MeV}}{0.5109989 \text{ MeV}} + 1 = 2.956951375 \approx 2.9569514.$$

(d) The corresponding speed parameter is

$$\beta = \sqrt{1 - \gamma^{-2}} = 0.941079236 \approx 0.94107924.$$