

••13 An electron is trapped in a one-dimensional infinite potential well that is 100 pm wide; the electron is in its ground state. What is the probability that you can detect the electron in an interval of width $\Delta x = 5.0$ pm centered at $x =$ (a) 25 pm, (b) 50 pm, and (c) 90 pm? (*Hint:* The interval Δx is so narrow that you can take the probability density to be constant within it.) **SSM WWW**

13. The probability that the electron is found in any interval is given by $P = \int |\psi|^2 dx$, where the integral is over the interval. If the interval width Δx is small, the probability can be approximated by $P = |\psi|^2 \Delta x$, where the wave function is evaluated for the center of the interval, say. For an electron trapped in an infinite well of width L , the ground state probability density is

$$|\psi|^2 = \frac{2}{L} \sin^2\left(\frac{\pi x}{L}\right), \quad P = \left(\frac{2\Delta x}{L}\right) \sin^2\left(\frac{\pi x}{L}\right).$$

(a) We take $L = 100$ pm, $x = 25$ pm, and $\Delta x = 5.0$ pm. Then,

$$P = \left[\frac{2(5.0 \text{ pm})}{100 \text{ pm}} \right] \sin^2 \left[\frac{\pi(25 \text{ pm})}{100 \text{ pm}} \right] = 0.050.$$

(b) We take $L = 100$ pm, $x = 50$ pm, and $\Delta x = 5.0$ pm. Then,

$$P = \left[\frac{2(5.0 \text{ pm})}{100 \text{ pm}} \right] \sin^2 \left[\frac{\pi(50 \text{ pm})}{100 \text{ pm}} \right] = 0.10, \quad P = \left[\frac{2(5.0 \text{ pm})}{100 \text{ pm}} \right] \sin^2 \left[\frac{\pi(90 \text{ pm})}{100 \text{ pm}} \right] = 0.0095.$$

••14 A particle is confined to the one-dimensional infinite potential well of Fig. 39-2. If the particle is in its ground state, what is its probability of detection between (a) $x = 0$ and $x = 0.25L$, (b) $x = 0.75L$ and $x = L$, and (c) $x = 0.25L$ and $x = 0.75L$?

14. We follow Sample Problem 39-3 in the presentation of this solution. The integration result quoted below is discussed in a little more detail in that Sample Problem. We note that the arguments of the sine functions used below are in radians.

(a) The probability of detecting the particle in the region $0 \leq x \leq L/4$ is

$$\left(\frac{2}{L}\right)\left(\frac{L}{\pi}\right)\int_0^{\pi/4} \sin^2 y \, dy = \frac{2}{\pi}\left(\frac{y}{2} - \frac{\sin 2y}{4}\right)\bigg|_0^{\pi/4} = 0.091.$$

(b) As expected from symmetry,

$$\left(\frac{2}{L}\right)\left(\frac{L}{\pi}\right)\int_{\pi/4}^{\pi} \sin^2 y \, dy = \frac{2}{\pi}\left(\frac{y}{2} - \frac{\sin 2y}{4}\right)\bigg|_{\pi/4}^{\pi} = 0.091.$$

(c) For the region $L/4 \leq x \leq 3L/4$, we obtain

$$\left(\frac{2}{L}\right)\left(\frac{L}{\pi}\right)\int_{\pi/4}^{3\pi/4} \sin^2 y \, dy = \frac{2}{\pi}\left(\frac{y}{2} - \frac{\sin 2y}{4}\right)\bigg|_{\pi/4}^{3\pi/4} = 0.82$$

which we could also have gotten by subtracting the results of part (a) and (b) from 1; that is, $1 - 2(0.091) = 0.82$.

••15 A one-dimensional infinite well of length 200 pm contains an electron in its third excited state. We position an electron-detector probe of width 2.00 pm so that it is centered on a point of maximum probability density. (a) What is the probability of detection by the probe? (b) If we insert the probe as described 1000 times, how many times should we expect the electron to materialize on the end of the probe (and thus be detected)? 

15. The position of maximum probability density corresponds to the center of the well:
 $x = L/2 = (200 \text{ pm})/2 = 100 \text{ pm}$.

(a) The probability of detection at x is given by Eq. 39-11:

$$p(x) = \psi_n^2(x) dx = \left[\sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L}x\right) \right]^2 dx = \frac{2}{L} \sin^2\left(\frac{n\pi}{L}x\right) dx$$

For $n = 3$, $L = 200 \text{ pm}$ and $dx = 2.00 \text{ pm}$ (width of the probe), the probability of detection at $x = L/2 = 100 \text{ pm}$ is

$$p(x = L/2) = \frac{2}{L} \sin^2\left(\frac{3\pi}{L} \cdot \frac{L}{2}\right) dx = \frac{2}{L} \sin^2\left(\frac{3\pi}{2}\right) dx = \frac{2}{L} dx = \frac{2}{200 \text{ pm}} (2.00 \text{ pm}) = 0.020.$$

(b) With $N = 1000$ independent insertions, the number of times we expect the electron to be detected is

$$n = Np = (1000)(0.020) = 20.$$

••16 An electron is in a certain energy state in a one-dimensional, infinite potential well from $x = 0$ to $x = L = 200$ pm. The electron's probability density is zero at $x = 0.300L$, and $x = 0.400L$; it is not zero at intermediate values of x . The electron then jumps to the next lower energy level by emitting light. What is the change in the electron's energy?

16. From Eq. 39-11, the condition of zero probability density is given by

$$\sin\left(\frac{n\pi}{L}x\right) = 0 \Rightarrow \frac{n\pi}{L}x = m\pi$$

where m is an integer. The fact that $x = 0.300L$ and $x = 0.400L$ have zero probability density implies

$$\sin(0.300n\pi) = \sin(0.400n\pi) = 0$$

which can be satisfied for $n = 10m$, where $m = 1, 2, \dots$. However, since the probability density is non-zero between $x = 0.300L$ and $x = 0.400L$, we conclude that the electron is in the $n = 10$ state. The change of energy after making a transition to $n' = 9$ is then equal to

$$|\Delta E| = \frac{h^2}{8mL^2} (n^2 - n'^2) = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})^2}{8(9.11 \times 10^{-31} \text{ kg})(2.00 \times 10^{-10} \text{ m})^2} (10^2 - 9^2) = 2.86 \times 10^{-17} \text{ J}$$