MP19-L05 學號:_____ 姓名:_____ 系級:_____得分:_____ 總分_____

填充或計算題都必須寫出計算過程，並將答案選項填云躓獢C

Problem 1 (單選題)(a)

(單選題)____ 1 可嘗試次數= 2 分數= 2 The significance of $|\Psi|^2$ is($|\Psi|^2$ 的物理意義為何？): (A) probability (B) energy (C) probability density (D) energy density (E) wavelength

(單選題)____ 2 可嘗試次數= 2 分數= 2 $\Psi(x)$ is the wave function for a particle moving along the x axis. The probability that the particle is in the interval from x = a to x = b is given by($\Psi(x)$ 是沿著 x 軸運動的粒子之波函數。粒子位於 $x = a$ 至 $x = b$ 區間內的機率，由下列哪個式子給出？): (A) $\Psi(b) - \Psi(a)$ (B) $|\Psi(b)|/|\Psi(a)|$ (C) $|\Psi(b)|^2/|\Psi(a)|^2$ (D) $\int_a^b \Psi(x)dx$ (E) $\int_a^b |\Psi(x)|^2 dx$

(單選題)____ 3 可嘗試次數= 2 分數= 1 If a wave function Ψ for a particle moving along the x axis is "normalized" then(若沿著 x 軸運動的粒子之波函數 Ψ 已經「歸一化」，則下列何者成立？): (A) $\int |\Psi|^2 dt = 1$ (B) $\int |\Psi|^2 dx = 1$ (C) $\frac{\partial \Psi}{\partial x} = 1$ (D) $\frac{\partial \Psi}{\partial t} = 1$ (E) $|\Psi|^2 = 1$

(單選題)____ 4 可嘗試次數= 2 分數= 1 A particle is confined by finite potential energy walls to a one-dimensional trap from x = 0 to x = L. Its wave funciton in the region x > L has the form(一個粒子受到有限高位能障壁的束縛，被限制在 $x = 0$ 至 $x = L$ 的一維位能阱內。它在 $x > L$ 區域的波函數具有下列哪一種形式？): (A) $\Psi(x) = A\sin(kx)$ (B) $\Psi(x) = Ae^{kx}$ (C) $\Psi(x) = Ae^{-kx}$ (D) $\Psi(x) = Ae^{ikx}$ (E) $\Psi(x) = 0$

(單選題)____ 5 可嘗試次數= 2 分數= 2 A particle in a certain finite potential energy well can have any of five quantized energy values and no more. Which of the following would allow it to have any of six quantized energy levels(某個有限深位能阱中的粒子，只能具有五個量子化的束縛態能量值。下列哪一項改變，可以使這個位能阱容許六個量子化的束縛態能階？)? (A) Increase the momentum of the particle (B) Decrease the momentum of the particle (C) Decrease the well width (D) Increase the well depth (E) Decrease the well depth

(單選題)____ 6 可嘗試次數= 2 分數= 2 An electron with energy E is incident upon a potential energy barrier of height $U > E$ and thickness L. The transmission coefficient T(一個能量為 E 的電子，入射到高度為 $U > E$、厚度為 L 的位能障壁。穿透係數 T 如何隨 L 改變？): (A) is zero (B) decreases exponentially with L (C) is proportional to 1/L (D) is proportional to 1/L2 (E) is non-zero and independent of L

(單選題)____ 7 可嘗試次數= 2 分數= 2 Identical particles, each with energy E, are incident on the following four potential energy barriers:
1. barrier height = 5E, barrier width = 2L
2. barrier height = 10E, barrier width = L
3. barrier height = 17E, barrier width = L/2
4. barrier height = 26E, barrier width = L/3

Rank the barriers in terms of the probability that the particles tunnel through them, from least probability to greatest probability.[相同種類的粒子，每個粒子的能量均為 E，分別入射到以下四個位能障壁(障壁高度 U | 障壁寬度 d)：請依粒子穿隧機率，由小到大排列。] (A) 1, 2, 3, 4 (B) 4, 3, 2, 1 (C) 1 and 2 tied, then 3, then4 (D) 2, then 2 and 3 tied, then 4 (E) 3, 2, 1, 4 (F)這個量越大，指數衰減越強，穿透機率越小。

Problem 2

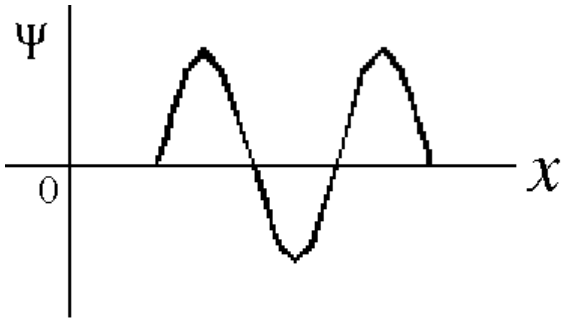
An electron is trapped in a deep well with a width of 0.3 nm. If it is in the state with quantum number n = 3 its kinetic energy is [一個電子被限制在寬度為 0.3 nm 的深位能阱中。若電子處於量子數 $n = 3$ 的狀態，求其動能。]: (01小題) 08 KE=_____ J 分數= 5

Problem 3

An electron in an atom initially has an energy 5.5 eV above the ground state energy. It drops to a state with energy 3.2 eV above the ground state energy and emits a photon in the process. Determine the wave length associated with the photon. [原子中的一個電子，最初的能量比基態高 5.5 eV。它躍遷到比基態高 3.2 eV 的狀態，並放出一個光子。求此光子的波長。] (01小題) 09 λ =_____ m 分數= 5

Problem 4

An electron is in a one-dimensional trap with zero potential energy in the interior and infinite potential energy at the walls. A graph of its wave function $\Psi(x)$ versus x is shown. The value of quantum number n [一個電子被限制在一維位能阱中，阱內位能為零，兩側阱壁的位能為無限大。圖中顯示電子的波函數 $\Psi(x)$ 隨位置 x 的變化。求其量子數 n]



=____ (01小題) 10 n=____ 分數= 5

Problem 5

PART.1 The Schrodinger equation of the one-dimensional simple harmonic oscillator with mass m and elastic force constant k can be written as:

$$\frac{d^2\Psi}{dA^2}+\frac{2B}{C^2}(P-\frac{1}{2}QR^2)\Psi=0.$$

Use the set of variables {m,k,x,E(=energy),hbar(=ħ)} for A,B,C,P,Q and R.

[一維簡諧振子的定態薛丁格方程式，經過變數變換後，可以化為： $\frac{d^2\Psi}{dy^2}+(\alpha-y^2)\Psi=0$ 其中：

$\alpha=\frac{2A}{B\omega}$, $\omega=\sqrt{\frac{k}{m}}$, $y=\sqrt{\frac{mP}{Q}}R$ 請從 $\{m,\omega,x,E,\hbar\}$ 中選出適當的物理量，填入 A、B、P、Q、R，並推導上述方程式。] (06小題) 11 A=___ . pick up your answer from the set:{m, k, x, E, hbar} 分數= 2 12 B=___. pick up your answer from the set:{m, k, x, E, hbar} 分數= 2 13 C=___. pick up your answer from the set:{m, k, x, E, hbar} 分數= 2 14 P=___. pick up your answer from the set:{m, k, x, E, hbar} 分數= 2 15 Q=___. pick up your answer from the set:{m, k, x, E, hbar} 分數= 2 16 R=___. pick up your answer from the set:{m, k, x, E, hbar} 分數= 2

Problem 5

PART.2 From previous problem, the above equation can be simplied as:

$$\frac{d^2\Psi}{dy^2}+(\alpha-y^2)\Psi=0$$

, where $\alpha=\frac{2A}{B\omega}$, $\omega=\sqrt{k/m}$ and $y=\sqrt{\frac{mP}{Q}}R$. Enter A,B and P,Q,R with {x,omega=ω,E,hbar}.

[承接 PART.1，一維簡諧振子的定態薛丁格方程式，經過變數變換後可化為： $\frac{d^2\Psi}{dy^2}+(\alpha-y^2)\Psi=0$ 其中： $\alpha=\frac{2A}{B\omega}$, $\omega=\sqrt{\frac{k}{m}}$, $y=\sqrt{\frac{mP}{Q}}R$ 請從 $\{x,\omega,E,\hbar\}$ 中選出適當的物理量，填入 A、B、P、Q、R，並推導上述方程式。(物理量可重複使用。)] (05小題) 17 A=___. pick up your answer from the set:{m, omega, x, E, hbar} 分數= 2 18 B=___. pick up your answer from the set:{m, omega,

x, E, hbar} 分數= 2 19 P=___. pick up your answer from the set:{m, omega, x, E, hbar} 分數= 2 20 Q=___. pick up your answer from the set:{m, omega, x, E, hbar} 分數= 2 21 R=___. pick up your answer from the set:{m, omega, x, E, hbar} 分數= 2

Problem 5

Part.3 From previous problem, find the eigen-energies of a quantum oscillator. [承接前題，求量子簡諧振子的能量本徵值 E_n] (01小題) 22 E_n=_____ [hbar, omega, m] 分數= 6

Problem 6
Consider the ground state wave function for a particle in an infinite potential well that extends from $x = 0$ to $x = L$. What is the probability of finding the particle from $x = L/4$ to $3L/4$?
[一個粒子處於範圍由 $x = 0$ 至 $x = L$ 的一維無限深位能阱之基態。求粒子出現在 $x = L/4$ 至 $x = 3L/4$ 區間內的機率。] (01小題) 23 probability=____ 分數= 8

Problem 6
Following previous problem, calculate the expectation value $< p >, < p^2 >$ for the ground state of the one-dimensional infinite potential well of length L. [承接前題，對長度為 L、範圍為 \(0

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24 $< p >$ = _____ 分數= 6

25 $< p^2 >$=____ [h,L,m] 分數= 6

Problem 7

An electron with a kinetic energy of $0.500\,\text{eV}$ is incident on a rectangular potential energy barrier of height $2.50\,\text{eV}$ and width $0.150\,\text{nm}$. The probability that the electron tunnels through the barrier can be estimated using the following exponential approximation: $T \approx e^{-2\kappa L}, \qquad \kappa = \dfrac{\sqrt{2m_e(V_0 - E)}}{\hbar}$ Here, m_e is the electron mass, E is the electron energy, V_0 is the barrier height, and L is the barrier width. (a) Using the exponential approximation above, calculate the probability that the electron tunnels through the barrier. (b) If the barrier width is doubled while its height remains unchanged, calculate the new tunneling probability. (c) If the electron's kinetic energy increases to $1.50\,\text{eV}$ while the barrier height and width remain at their original values, calculate the new tunneling probability. Explain why the electron can tunnel through the barrier even when its energy is lower than the barrier height.

[一個動能為 $0.500\,\text{eV}$ 的電子，入射到高度為 $2.50\,\text{eV}$、寬度為 $0.150\,\text{nm}$ 的矩形位能障壁。 電子穿透障壁的機率可利用下列指數近似公式估計：

$$T \approx e^{-2\kappa L}, \qquad \kappa = \frac{\sqrt{2m_e(V_0 - E)}}{\hbar}$$

其中 m_e 為電子質量，(E) 為電子能量，V_0 為位能障壁高度，L 為障壁寬度。 (a) 利用上述指數近似公式，計算電子穿透障壁的機率。 (b) 如果障壁寬度增加為原來的兩倍，而障壁高度保持不變，求新的穿透機率。 (c) 如果電子的動能增加至 $1.50\,\text{eV}$，而障壁高度與寬度維持原來的數值，求新的穿透機率。為什麼電子的能量小於位能障壁高度時，仍然可能穿透障壁。] (03小題) 26 (a)電子穿透障壁的機率=_____ % 分數= 3 27 (b)障壁寬度增加為原來的兩倍，新的穿透機率=_____ % 分數= 3 28 (c)電子的動能增加至 $1.50\,\text{eV}$，新的穿透機率=_____ % 分數= 3