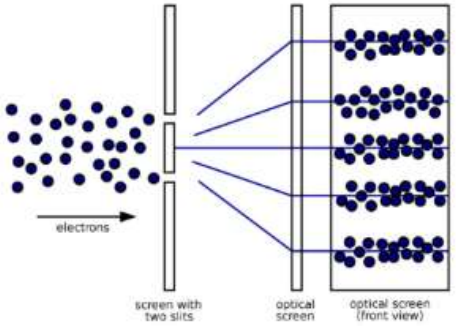
MP19-L03A

MP19-L03

2.干涉條紋，說明電子的傳播具有波動特性

__C__ 1 可嘗試次數= 2 分數= 1　1927年，柯林頓·戴維森與雷斯特·革末在貝爾實驗室將電子射向鎳結晶，發現其繞射圖譜，這個實驗結果證實了哪一個想法? (A)光具有粒子的特性　(B)光具有波動的特性　(C)粒子具有波的特性　(D)波具有粒子的特性　繞射是波動的重要特徵，因此這項實驗證實電子具有波動性。

__C__ 2 可嘗試次數= 2 分數= 1　1961年Claus Jonsson的電子雙狹縫實驗發現，電子只能走左邊或右邊這兩條路徑，最後抵達屏幕記錄下來，屏幕累積20分鐘(記錄了12000顆電子，如圖)，此實驗在2002年被Physics World雜誌讀者選為最美麗的物理實驗。這個實驗證實了 (A)電子滿足牛頓運動定律　(B)電子滿足狹義相對論的運動描述　(C)電子具有物質波的特性　(D)電子發生輻射現象　(E)電子的角動量空間量子化



__C__ 3 可嘗試次數= 2 分數= 1　根據德布洛依物質波理論，一個質量為m，動量為p，的質點，其質波的波長為下列哪一個公式? (A)$\lambda = hp$　(B)$\lambda = p/h$　(C)$\lambda = h/p$　(D)$\lambda = \frac{mh}{p^2}$　(E)$\lambda = \frac{h^2}{mp}$

__B__ 4 可嘗試次數= 2 分數= 1　一個帶電質點它的質量是m，電量q，最初靜止。受電位差V加速後，求德布羅意波長=? (A)$\sqrt{2mqV}$　(B)$\frac{h}{\sqrt{2mqV}}$　(C)$h\sqrt{2mqV}$　(D)$\frac{2h}{\sqrt{mqV}}$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$$

$$\frac{1}{2}mv^2 = qV \qquad v = \sqrt{\frac{2qV}{m}} \qquad p = mv = m\sqrt{\frac{2qV}{m}} = \sqrt{2mqV}$$

MP19-L03A

1. 1927 年，Davisson–Germer利用電子束照射鎳晶體，觀察到電子繞射現象。

__A__ 5 可嘗試次數= 2 分數= 1 Evidence for the wave nature of matter is[物質波動性的證據是]: (A) electron diffraction experiments of Davisson and Germer (B) Thompson's measurement of e/m (C) Young's double slit experiment (D) the Compton effect (E) Lenz's law

$K = \frac{p^2}{2m}$ $p = \sqrt{2mK}$ $\lambda = \frac{h}{\sqrt{2mK}}$ $m_p \approx 1836 m_e$

__B__ 6 可嘗試次數= 2 分數= 1 Consider the following three particles: 1. a free electron with kinetic energy K 2. a free proton with kinetic energy K 3. a free proton with kinetic energy 2K. Rank them according to the wavelengths of their waves, least to greatest.[考慮以下三種粒子：1. 動能為 K 的自由電子 2. 動能為 K 的自由質子 3. 動能為 2K 的自由質子 依其波的波長從小到大排序。] (A) 1, 2, 3 (B) 3, 2, 1 (C) 2, 3, 1 (D) 1, 3, 2 (E) 1, then 2 and 3 tied

__B__ 7 可嘗試次數= 2 分數= 1 A slowly moving proton has its velocity doubled. How does this affect its de Broglie wavelength[一個運動速度較慢的質子速度加倍。這會對其德布羅意波長產生什麼影響？]? (A)it is divided by 4 (B)it is divided by 2 (C)it is doubled (D) it is quadrupled

__B__ 8 可嘗試次數= 2 分數= 1 A slowly moving proton has its kinetic energy doubled. How does this affect its deBroglie wavelength[一個運動緩慢的質子的動能增加了一倍。這會對其德布羅意波長產生什麼影響？]? (A)it is divided by $\sqrt{4}$ (B)it is divided by $\sqrt{2}$ (C)it is multiplied by $\sqrt{2}$ (D)it is multiplied by $\sqrt{4}$

$K = \frac{p^2}{2m}$ $p = \sqrt{2mK}$ $\lambda = \frac{h}{\sqrt{2mK}}$

$K' = 2K$ $\lambda' = \frac{h}{\sqrt{2m(2K)}} = \frac{h}{\sqrt{4mK}}$

$\frac{\lambda'}{\lambda} = \frac{\sqrt{2mK}}{\sqrt{4mK}} = \frac{1}{\sqrt{2}}$

$\lambda = \frac{h}{mv}$ $v' = 2v$ $\lambda' = \frac{h}{m(2v)} = \frac{\lambda}{2}$

9. $\lambda = \frac{h}{\sqrt{2mK}}$ $\lambda \propto \frac{1}{\sqrt{m}}$ $m_p \approx 1836 m_e$

10. $\lambda = \frac{h}{p}$ $p_e = p_p$ $\lambda_e = \lambda_p$ $f = \frac{K}{h}$ $K = \frac{p^2}{2m}$ $f = \frac{p^2}{2mh}$ $\frac{f_e}{f_p} = \frac{m_p}{m_e} \approx 1836$

MP19-L03A

__A__ 9 可嘗試次數= 2 分數= 1 An electron and a proton have the same kinetic energy. Which has the greater de Broglie wavelength[電子和質子具有相同的動能，哪一個的德布羅意波長較長？]? (A)electron (B)proton (C)same

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__D__ 10 可嘗試次數= 2 分數= 1 A free electron and a free proton have the same momentum. This means that, compared to the matter wave associated with the proton, the matter wave associated with the electron has[自由電子和自由質子具有相同的動量。這意味著，與質子相關的物質波相比，電子相關的物質波具有]: (A) a shorter wavelength and a greater frequency (B) a longer wavelength and a greater frequency (C) the same wavelength and the same frequency (D) the same wavelength and a greater frequency (E) the same wavelength and a smaller frequency

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__D__ 11 可嘗試次數= 2 分數= 1 A free electron and a free proton have the same kinetic energy. This means that, compared to the matter wave associated with the proton, the matter wave associated with the electron has[自由電子和自由質子具有相同的動能。這意味著，與質子相關的物質波相比，電子相關的物質波具有較小的動能。]: (A) a shorter wavelength and a greater frequency (B) a longer wavelength and a greater frequency (C) a shorter wavelength and the same frequency (D) a longer wavelength and the same frequency (E) a shorter wavelength and a smaller frequency

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__E__ 12 可嘗試次數= 2 分數= 1 A non-relativistic free electron has kinetic energy K. If its wavelength doubles, its kinetic energy is[非相對論性自由電子的動能為K。如果它的波長加倍，則其動能變成]: (A) 4 K (B) 2 K (C) still K (D) K/2 (E) K/4

11. $\lambda = \frac{h}{\sqrt{2mK}}$ $\lambda \propto \frac{1}{\sqrt{m}}$ $m_e < m_p$ $\lambda_e > \lambda_p$ $f = \frac{K}{h}$ $K_e = K_p$ $f_e = f_p$

12. $K = \frac{h^2}{2m\lambda^2}$ $\lambda' = 2\lambda$ $K' = \frac{h^2}{2m(2\lambda)^2}$

MP19-L03A

$$E_n = \frac{n^2h^2}{8mL^2}, \quad n = 1, 2, 3, \ldots \qquad E_1 = \frac{h^2}{8mL^2} \quad E_3 = \frac{9h^2}{8mL^2} \qquad \frac{E_3}{E_1} = \frac{3^2}{1^2} = 9$$

__D__ 13 可嘗試次數= 2 分數= 1 An electron is in a one-dimensional trap with zero potential energy in the interior and infinite potential energy at the walls. The ratio E_3/E_1 of the energy for n = 3 to that for n = 1 is[一個電子處於一維勢阱中，勢阱內部位能為零，勢阱壁面位能為無限大。 n = 3 時的能量與 n = 1 時的能量之比 E_3/E_1 為]: (A) 1/3 (B) 1/9 (C) 3/1 (D) 9/1 (E) 1/1

$E_3 - E_1 = (9-1)E_1 = 8E_1$ $E_3 - E_2 = (9-4)E_1 = 5E_1$ $E_4 - E_3 = (16-9)E_1 = 7E_1$

__C__ 14 可嘗試次數= 2 分數= 1 A particle is trapped in a one-dimensional well with infinite potential energy at the walls. Three possible pairs of energy levels are: 1. n = 3 and n = 1 2. n = 3 and n = 2 3. n = 4 and n = 3 Order these pairs according to the difference in energy. least to greatest.[一個粒子被束縛在一個一維位勢阱中，勢阱壁面的位能為無窮大。可能存在三對能階： 1. n = 3 和 n = 1 2. n = 3 和 n = 2 3. n = 4 和 n = 3 請依能量差從小到大的順序排列這三對能階。] (A) 1, 2, 3 (B) 3, 2, 1 (C) 2, 3, 1 (D) 1, 3, 2 (E) 3, 1, 2

__D__ 15 可嘗試次數= 2 分數= 1 Identical particles are trapped in one-dimensional wells with infinite potential energy at the walls. The widths L of the traps and the quantum numbers n of the particles are (1). $L = 2L_0, n = 2$, (2). $L = 2L_0, n = 4$, (3). $L = 3L_0, n = 3$, (4). $L = 4L_0, n = 2$. Rank them according to the kinetic energies of the particles, least to greatest. [相同的粒子被束縛在一維勢阱中，勢阱壁具有無限大的位能。勢阱的寬度 L 和粒子的量子數 n 分別為：(1). $L = 2L_0, n = 2$, (2). $L = 2L_0, n = 4$, (3). $L = 3L_0, n = 3$, (4). $L = 4L_0, n = 2$. 。請根據粒子的動能從小到大對它們進行排序。] (A) 1, 2, 3, 4 (B) 4, 3, 2, 1 (C) 1 and 3 tied, then 2, 4 (D) 4, 2, then 1 and 3 tied (E) 1, 3, then 2 and 4 tied

1	$2L_0$	2	K_0	3	$3L_0$	3	K_0
2	$2L_0$	4	$4K_0$	4	$4L_0$	2	$K_0/4$

$$K_{(4)} < K_{(1)} = K_{(3)} < K_{(2)}$$

MP19-L03A

Problem 2

PART.1 Use the theory of special relativity to derive the following formula for the momentum of a particle in terms of its kinetic energy (K), mass (m) and speed of light c: p^2c^2=_____ .

利用狹義相對論推導出粒子動量與其動能 (K)、質量 (m) 和光速 c 之間的以下公式：p^2c^2=_____ (01小題)

---

p^2c^2=_____ [K,m,c]

16: **ANS:=k*(K+2*m*c**2)**

$$E^2 = p^2c^2 + m^2c^4$$

E：粒子的總能量

p：相對論動量

m：粒子的靜止質量

c：真空中的光速

K：粒子的動能

粒子的總能量等於靜止能量與動能之和：

$$E = K + mc^2$$

$$(K + mc^2)^2 = p^2c^2 + m^2c^4$$

$$K^2 + 2Kmc^2 + m^2c^4 = p^2c^2 + m^2c^4$$

$$K^2 + 2Kmc^2 = p^2c^2$$

$$p^2c^2 = K^2 + 2Kmc^2 = K(K + 2mc^2)$$

$$p = \frac{\sqrt{K^2 + 2Kmc^2}}{c}$$

MP19-L03A

PART.2 Following previous problem, derive a relativistically correct formula that gives the de Broglie wavelength of a charged particle in terms of the charge q, mass m and the potential difference V through which it has been accelerated. λ =_____ [承接上題，推導出一個相對論意義上的公式，該公式以電荷 q、質量 m 和粒子加速所經過的電位差 V 表示帶電粒子的德布羅意波長 λ。 λ = _____] (01小題)

λ=_____ [h,q,V,m,c]

17: **ANS:=h/sqrt(2*q*V*m+(q**2*V**2)/c**2))**

$$K = qV$$

$$p^2c^2 = K^2 + 2Kmc^2$$

$$p^2c^2 = (qV)^2 + 2mqVc^2$$

$$p = \frac{\sqrt{(qV)^2 + 2mqVc^2}}{c}$$

$$\lambda = \frac{h}{p}$$

$$\lambda = \frac{hc}{\sqrt{(qV)^2 + 2mqVc^2}}$$

$$\lambda = \frac{h}{\sqrt{2mqV + \frac{q^2V^2}{c^2}}}$$

$$\lambda = \frac{h}{\sqrt{2mqV\left(1 + \frac{qV}{2mc^2}\right)}}$$

含相對論修正的德布羅意波長公式

非相對論極限

$$qV \ll mc^2$$

$$\frac{qV}{2mc^2} \ll 1$$

$$1 + \frac{qV}{2mc^2} \approx 1$$

$$\lambda \approx \frac{h}{\sqrt{2mqV}}$$

MP19-L03A

Problem 3

投手將一顆150克的棒球以時速144公里/小時丟出，物質波的波長約為__公尺? (01小題)

$\lambda =$_____ m

18: **ANS:** **E-34**

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$m = 150\ \text{g} = 0.150\ \text{kg}$$

$$v = 144\ \text{km/h} = \frac{144}{3.6} = 40.0\ \text{m/s}$$

$$h = 6.626 \times 10^{-34}\ \text{J} \cdot \text{s}$$

$$p = mv = 0.150 \times 40.0 = 6.00\ \text{kg} \cdot \text{m/s}$$

計算德布羅意波長：

$$\lambda = \frac{6.626 \times 10^{-34}}{0.150 \times 40.0}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{6.00}$$

$$\lambda = 1.10 \times 10^{-34}\ \text{m}$$

正確答案：

$$\lambda \approx 1.1 \times 10^{-34}\ \text{m}$$

這個波長遠小於原子核的典型尺度(約 10^{-15} m)，因此在一般實驗條件下，無法觀察到棒球的物質波繞射或干涉現象。

MP19-L03A

Problem 4

如果一個電子和一個光子的波長都是2.0Å，請計算(a)電子的動量, (b)光子的動量, (c)電子的能量, (d)光子的能量 (04小題)

(a)_____ kg.m/s

19: **ANS:=3.31E-24**

(b)_____ kg.m/s

20: **ANS:=3.31E-24**

(c)_____ eV

21: **ANS:=37.8**

(d)_____ eV

22: **ANS:=6200**

$\lambda = 2.0\ \text{Å} = 2.0 \times 10^{-10}\ \text{m}$

$h = 6.626 \times 10^{-34}\ \text{J} \cdot \text{s}$

$m_e = 9.109 \times 10^{-31}\ \text{kg}$

$c = 3.00 \times 10^{8}\ \text{m/s}$

$1\ \text{eV} = 1.602 \times 10^{-19}\ \text{J}$

(a) 電子的動量

根據德布羅意關係式：

$$\lambda = \frac{h}{p}$$

$$p_e = \frac{h}{\lambda}$$

代入數值：

$$p_e = \frac{6.626 \times 10^{-34}}{2.0 \times 10^{-10}}$$

$$p_e = 3.31 \times 10^{-24}\ \text{kg} \cdot \text{m/s}$$

(b) 光子的動量

$$p_\gamma = \frac{h}{\lambda}$$

由於電子與光子的波長相同：

$$p_\gamma = p_e$$

$$p_\gamma = 3.31 \times 10^{-24}\ \text{kg} \cdot \text{m/s}$$

(c) 電子的動能

電子的非相對論動能：

$$K_e = \frac{p_e^2}{2m_e}$$

$$K_e = \frac{h^2}{2m_e\lambda^2}$$

$$K_e = \frac{(6.626 \times 10^{-34})^2}{2(9.109 \times 10^{-31})(2.0 \times 10^{-10})^2}$$

$$K_e = 6.02 \times 10^{-18}\ \text{J}$$

$$= \frac{6.02 \times 10^{-18}}{1.602 \times 10^{-19}}\ \text{eV}$$

(d) 光子的能量

$$E_\gamma = hf \qquad f = \frac{c}{\lambda}$$

$$E_\gamma = \frac{hc}{\lambda}$$

$$E_\gamma = \frac{(6.626 \times 10^{-34})(3.00 \times 10^{8})}{2.0 \times 10^{-10}}$$

$$E_\gamma = 9.94 \times 10^{-16}\ \text{J}$$

$$= \frac{9.94 \times 10^{-16}}{1.602 \times 10^{-19}}\ \text{eV}$$

$$\approx 6.20 \times 10^{3}\ \text{eV} = 6.20\ \text{keV}$$

MP19-L03A

Problem 5

Find the de Broglie wavelength of (a) an electron whose speed is 1.0×10^8 m/s, and (b) an electron whose speed is 2.0×10^8 m/s.

求 (a) 速度為 1.0×10^8 m/s 的電子的德布羅意波長，以及 (b) 速度為 2.0×10^8 m/s 的電子的德布羅意波長。題)

(a) $\lambda =$_____ pm

23: **ANS:=6.86**

(b) $\lambda =$_____ pm

24: **ANS:=2.71**

$h = 6.626 \times 10^{-34}\ \text{J} \cdot \text{s}$

$m_e = 9.109 \times 10^{-31}\ \text{kg}$

$c = 3.00 \times 10^8\ \text{m/s}$

$1\ \text{pm} = 10^{-12}\ \text{m}$

由於電子的速度已接近光速，必須考慮**狹義相對論修正**。

根據德布羅意關係式： $\lambda = \dfrac{h}{p}$

$$p = \gamma m v = \frac{mv}{\sqrt{1 - v^2/c^2}}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\lambda = \frac{h}{\gamma m v} = \frac{h}{mv}\sqrt{1 - \frac{v^2}{c^2}}$$

(a) 電子速度為 1.0×10^8 m/s

首先計算勞侖茲因子：

$$\gamma = \frac{1}{\sqrt{1 - (1.0/3.0)^2}} = 1.06066$$

$$p = \gamma m_e v$$

$$p = (1.06066)(9.109 \times 10^{-31})(1.0 \times 10^8)$$

$$p = 9.66 \times 10^{-23}\ \text{kg} \cdot \text{m/s}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{9.66 \times 10^{-23}} = 6.86 \times 10^{-12}\ \text{m}$$

(b) 電子速度為 2.0×10^8 m/s

計算勞侖茲因子：

$$\gamma = \frac{1}{\sqrt{1 - (2.0/3.0)^2}} = \frac{1}{\sqrt{5/9}} = 1.34164$$

$$p = \gamma m_e v$$

$$p = (1.34164)(9.109 \times 10^{-31})(2.0 \times 10^8)$$

$$p = 2.44 \times 10^{-22}\ \text{kg} \cdot \text{m/s}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{2.44 \times 10^{-22}}$$

$$\lambda = 2.71 \times 10^{-12}\ \text{m}$$

MP19-L03A

Problem 6

Find the kinetic energy of an electron whose de Broglie wavelength is the same as that of a 200-keV x-ray in (a)non-relativistic and (b)relativistic calculations.

求德布羅意波長與 200 keV X 射線相同的電子的動能，分別採用（a）非相對論計算和（b）相對論計算。(02小題)

(a)non-relativistic KE=____ keV

25: **ANS:=39.14**

(a)relativistic KE=____ keV

26: **ANS:=37.74**

X 射線光子的能量：

$$E_\gamma = 200 \text{ keV}$$

電子的靜止能量：

$$m_e c^2 = 511 \text{ keV}$$

由於電子與光子的波長相同：

$$\lambda_e = \lambda_\gamma$$

$$p = \frac{h}{\lambda} \qquad p_e = p_\gamma$$

$$E_\gamma = p_\gamma c \qquad p_e c = 200 \text{ keV}$$

電子的非相對論動能：

$$K_{\text{NR}} = \frac{p_e^2}{2m_e}$$

分子與分母同乘 c^2：

$$K_{\text{NR}} = \frac{p_e^2 c^2}{2m_e c^2}$$

代入數值：

$$K_{\text{NR}} = \frac{(200 \text{ keV})^2}{2(511 \text{ keV})}$$

$$K_{\text{NR}} = \frac{40000}{1022} \text{ keV}$$

$$K_{\text{NR}} = 39.1 \text{ keV}$$

根據狹義相對論的能量 - 動量關係：

$$E^2 = p_e^2 c^2 + m_e^2 c^4$$

電子的總能量 $E = K_{\text{R}} + m_e c^2$

$$(K_{\text{R}} + m_e c^2)^2 = p_e^2 c^2 + m_e^2 c^4$$

$$K_{\text{R}} = \sqrt{p_e^2 c^2 + m_e^2 c^4} - m_e c^2$$

代入數值：

$$K_{\text{R}} = \sqrt{(200)^2 + (511)^2} - 511 \text{ keV}$$

$$K_{\text{R}} = \sqrt{301121} - 511 \text{ keV}$$

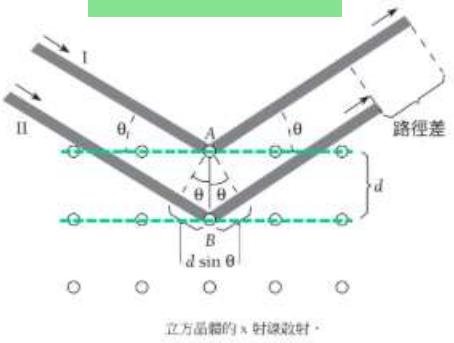
$$K_{\text{R}} = 548.745 - 511 \text{ keV}$$

$$K_{\text{R}} \approx 37.7 \text{ keV}$$

MP19-L03A

Problem 7

A beam of neutrons emerging from a nuclear reactor contains neutrons with a variety of energies. To select neutrons with a kinetic energy of 0.050 eV, the beam is directed onto a crystal whose atomic planes are separated by 0.20 nm. Using Bragg's law, calculate the smallest and largest possible Bragg angles between 0° and 90°. The Bragg angle is defined as the angle between the incident neutron beam and the crystal planes.



從核反應器射出的中子束包含各種不同能量的中子。為了選出動能為 0.050 eV 的中子，讓中子束入射至原子晶面間距為 0.20 nm 的晶體。 根據布拉格定律，計算 0° 至 90° 之間可能產生繞射的最小及最大布拉格角。布拉格角定義為入射中子束與晶體原子晶面之間的夾角。 (02小題)

(a)smallest angle, θ =_____ degree

27: **ANS:=18.7**

(b)largest angle, θ =_____ degree

28: **ANS:=73.6**

中子的動能： $K = 0.050\ \text{eV}$

晶面間距： $d = 0.20\ \text{nm}$

中子質量：$1.675 \times 10^{-27}\ \text{kg}$

$h = 6.626 \times 10^{-34}\ \text{J} \cdot \text{s}$

$$K = \frac{p^2}{2m_n} \qquad \lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_nK}}$$

$$K = 0.050(1.602 \times 10^{-19})$$

動能遠小於靜止能量

非相對論

$$= 8.01 \times 10^{-21}\ \text{J}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{\sqrt{2(1.675 \times 10^{-27})(8.01 \times 10^{-21})}}$$

$$= 1.28 \times 10^{-10}\ \text{m} = 0.128\ \text{nm}$$

$$n\lambda = 2d\sin\theta$$

$$\sin\theta = \frac{n(0.128)}{0.40}$$

$$n = 1, 2, 3$$

最小布拉格角： n=1

$$\theta_{\min} = \sin^{-1}\left(\frac{0.128}{0.40}\right) \approx 18.7°$$

最大布拉格角： n=3

$$\theta_{\max} = \sin^{-1}\left(\frac{3(0.128)}{0.40}\right) \approx 73.6°$$

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Problem 8

Obtain an expression for the energy levels (in MeV) of a neutron confined to a one-dimensional box 1.00×10^{-14} m wide. What is the neutron's minimum energy? (The diameter of an atomic nucleus is of this order of magnitude.)

求出限制在邊長為 1.00×10^{-14} 公尺的一維盒子中的中子的能階表達式（單位：MeV）。中子的最小能量是多少？（原子核的直徑與此數量級相同。） (01小題)

KE_{min}=_____ Mev

29: **ANS:=2.05**

位能井寬度：

$L = 1.00 \times 10^{-14}\ \text{m}$

中子質量：

$m_n = 1.675 \times 10^{-27}\ \text{kg}$

$h = 6.626 \times 10^{-34}\ \text{J} \cdot \text{s}$

$1\ \text{MeV} = 1.602 \times 10^{-13}\ \text{J}$

對於一維無限深位能井，粒子的波函數必須滿足邊界條件：

$\psi(0) = \psi(L) = 0$ 形成特定波長的駐波：

$$L = n\frac{\lambda_n}{2}, \qquad n = 1, 2, 3, \ldots \qquad \lambda_n = \frac{2L}{n}$$

$$p_n = \frac{h}{\lambda_n} = \frac{nh}{2L} \qquad K_n = \frac{p_n^2}{2m_n}$$

$$K_n = \frac{1}{2m_n}\left(\frac{nh}{2L}\right)^2 = \frac{n^2h^2}{8m_nL^2}, \qquad n = 1, 2, 3, \ldots$$

由於基態動能約為 2.05 MeV，遠小於中子的靜止能量約 940 MeV，因此使用非**相對論**動能公式是合理的。

MP19-L03A

Problem 9

A proton in a one-dimensional box has an energy of 400 keV in its first excited state. How wide is the box? Use non-relativistic consideration.

一維盒子中的質子在第一激發態的能量為 400 keV。這個盒子有多寬？請使用非相對論方法考慮。 (04小題)

(a)Obtain the width by the exact formula for the energy. The width of the box[利用能量的精確公式求得寬度。盒子的寬度]=_____ m

30: **ANS:=4.53E-14**

$$E_n = n^2 E_1 \qquad E_2 = 4E_1$$

$$E_1 = \frac{E_2}{4} = \frac{400\ \text{keV}}{4} = 100\ \text{keV}$$

$$E_n = \frac{n^2 h^2}{8 m_p L^2} \qquad L = \frac{h}{\sqrt{2 m_p E_2}}$$

$$L = \frac{6.626 \times 10^{-34}}{\sqrt{2(1.673 \times 10^{-27})(6.408 \times 10^{-14})}}$$

$$= 4.53 \times 10^{-14}\ \text{m}$$

(b)the ground state energy E_1=_____ eV

31: **ANS:=1.0E5**

(c)the momentum of the proton in the ground state, p_1=_____ kg.m/s

32: **ANS:=7.311E-21**

(d)Use uncertainty principle and make the approximation that $\Delta p \simeq p_1$ to estimate the minmum width[利用不確定原理，並近似假設 $\Delta p \simeq p_1$ 來估計最小寬度。]=__ m

33: **ANS:=7.211E-15**

$$\Delta x\, \Delta p \geq \frac{\hbar}{2} \qquad \Delta p \simeq p_1$$

$$\Delta x_{\min} \simeq \frac{\hbar}{2p_1} \simeq \frac{1.055 \times 10^{-34}}{2(7.32 \times 10^{-21})}$$

$$\simeq 7.21 \times 10^{-15}\ \text{m}$$

$$E_1 = \frac{p_1^2}{2m_p} \qquad p_1 = \sqrt{2 m_p E_1}$$

$$E_1 = (1.00 \times 10^5)(1.602 \times 10^{-19}) = 1.602 \times 10^{-14}\ \text{J}$$

$$p_1 = \sqrt{2(1.673 \times 10^{-27})(1.602 \times 10^{-14})}$$

$$= 7.32 \times 10^{-21}\ \text{kg} \cdot \text{m/s}$$

MP19-L03A

Problem 10

A proton has a kinetic energy of 1.0 MeV. If its momentum is measured with an uncertainty of 5.0%, what is the minimum uncertainty in its position?

一個質子的動能為 1.0 MeV。如果它的動量測量不確定度為 5.0%，那麼它的位置的最小不確定度是多少？ (01小題)

Δx=_____

34: **ANS:=4.56E-14**

由於質子的動能 1.0 MeV 遠小於其靜止能量約 938 MeV，可以使用非相對論

質子動能： $K = 1.0\ \text{MeV} = 1.602 \times 10^{-13}\ \text{J}$

質子質量： $m_p = 1.673 \times 10^{-27}\ \text{kg}$

$\hbar = 1.055 \times 10^{-34}\ \text{J} \cdot \text{s}$

$$K = \frac{p^2}{2m_p} \qquad p = \sqrt{2m_pK}$$

$$p = \sqrt{2(1.673 \times 10^{-27})(1.602 \times 10^{-13})}$$

$$p = 2.315 \times 10^{-20}\ \text{kg} \cdot \text{m/s}$$

動量的相對不確定度：

$$\frac{\Delta p}{p} = 5.0\% = 0.050$$

$$\Delta p = 0.050p$$

$$\Delta p = 0.050(2.315 \times 10^{-20})$$

$$\Delta p = 1.158 \times 10^{-21}\ \text{kg} \cdot \text{m/s}$$

$$\Delta x\,\Delta p \geq \frac{\hbar}{2} \qquad \Delta x_{\min} = \frac{\hbar}{2\Delta p}$$

$$\Delta x_{\min} = \frac{1.055 \times 10^{-34}}{2(1.158 \times 10^{-21})}$$

$$= 4.56 \times 10^{-14}\ \text{m}$$