

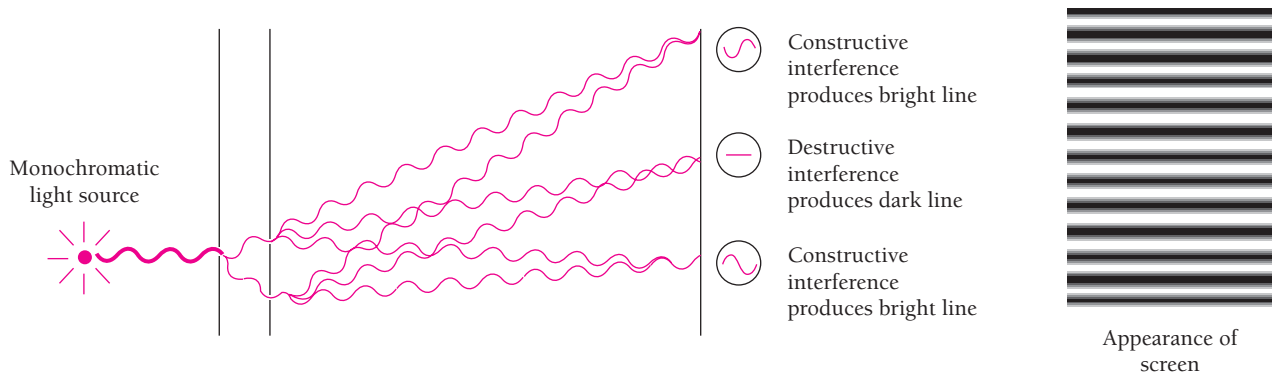


Figure 2.4 Origin of the interference pattern in Young's experiment. Constructive interference occurs where the difference in path lengths from the slits to the screen is $0, \lambda, 2\lambda, \dots$. Destructive interference occurs where the path difference is $\lambda/2, 3\lambda/2, 5\lambda/2, \dots$.

equal or differ by a whole number of wavelengths ($\lambda, 2\lambda, 3\lambda, \dots$), constructive interference occurs and a bright line is the result. At intermediate places the interference is only partial, so the light intensity on the screen varies gradually between the bright and dark lines.

Interference and diffraction are found only in waves—the particles we are familiar with do not behave in those ways. If light consisted of a stream of classical particles, the entire screen would be dark. Thus Young's experiment is proof that light consists of waves. Maxwell's theory further tells us what kind of waves they are: electromagnetic. Until the end of the nineteenth century the nature of light seemed settled forever.

2.2 BLACKBODY RADIATION

Only the quantum theory of light can explain its origin

Following Hertz's experiments, the question of the fundamental nature of light seemed clear: light consisted of em waves that obeyed Maxwell's theory. This certainty lasted only a dozen years. The first sign that something was seriously amiss came from attempts to understand the origin of the radiation emitted by bodies of matter.

We are all familiar with the glow of a hot piece of metal, which gives off visible light whose color varies with the temperature of the metal, going from red to yellow to white as it becomes hotter and hotter. In fact, other frequencies to which our eyes do not respond are present as well. An object need not be so hot that it is luminous for it to be radiating em energy; *all* objects radiate such energy continuously whatever their temperatures, though which frequencies predominate depends on the temperature. At room temperature most of the radiation is in the infrared part of the spectrum and hence is invisible.

The ability of a body to radiate is closely related to its ability to absorb radiation. This is to be expected, since a body at a constant temperature is in thermal equilibrium with its surroundings and must absorb energy from them at the same rate as it emits energy. It is convenient to consider as an ideal body one that absorbs *all* radiation incident upon it, regardless of frequency. Such a body is called a **blackbody**.

The point of introducing the idealized blackbody in a discussion of thermal radiation is that we can now disregard the precise nature of whatever is radiating, since

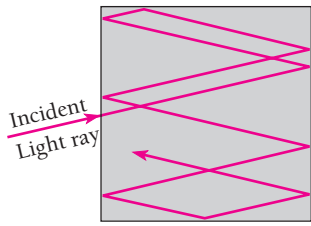


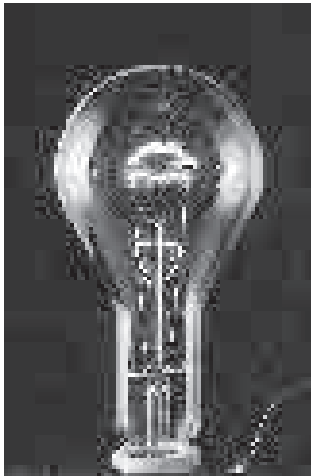
Figure 2.5 A hole in the wall of a hollow object is an excellent approximation of a blackbody.

all blackbodies behave identically. In the laboratory a blackbody can be approximated by a hollow object with a very small hole leading to its interior (Fig. 2.5). Any radiation striking the hole enters the cavity, where it is trapped by reflection back and forth until it is absorbed. The cavity walls are constantly emitting and absorbing radiation, and it is in the properties of this radiation (**blackbody radiation**) that we are interested.

Experimentally we can sample blackbody radiation simply by inspecting what emerges from the hole in the cavity. The results agree with everyday experience. A blackbody radiates more when it is hot than when it is cold, and the spectrum of a hot blackbody has its peak at a higher frequency than the peak in the spectrum of a cooler one. We recall the behavior of an iron bar as it is heated to progressively higher temperatures: at first it glows dull red, then bright orange-red, and eventually it becomes “white hot.” The spectrum of blackbody radiation is shown in Fig. 2.6 for two temperatures.

The Ultraviolet Catastrophe

Why does the blackbody spectrum have the shape shown in Fig. 2.6? This problem was examined at the end of the nineteenth century by Lord Rayleigh and James Jeans. The details of their calculation are given in Chap. 9. They started by considering the radiation inside a cavity of absolute temperature T whose walls are perfect reflectors to be a series of standing em waves (Fig. 2.7). This is a three-dimensional generalization of standing waves in a stretched string. The condition



The color and brightness of an object heated until it glows, such as the filament of this light bulb, depends upon its temperature, which here is about 3000 K. An object that glows white is hotter than it is when it glows red, and it gives off more light as well.

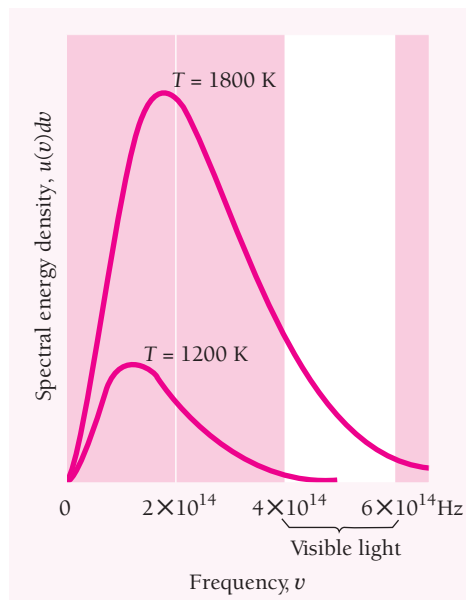


Figure 2.6 Blackbody spectra. The spectral distribution of energy in the radiation depends only on the temperature of the body. The higher the temperature, the greater the amount of radiation and the higher the frequency at which the maximum emission occurs. The dependence of the latter frequency on temperature follows a formula called Wien’s displacement law, which is discussed in Sec. 9.6.

for standing waves in such a cavity is that the path length from wall to wall, whatever the direction, must be a whole number of half-wavelengths, so that a node occurs at each reflecting surface. The number of independent standing waves $G(\nu)d\nu$ in the frequency interval between ν and $d\nu$ per unit volume in the cavity turned out to be

Density of standing waves in cavity

$$G(\nu)d\nu = \frac{8\pi\nu^2 d\nu}{c^3} \quad (2.1)$$

This formula is independent of the shape of the cavity. As we would expect, the higher the frequency ν , the shorter the wavelength and the greater the number of possible standing waves.

The next step is to find the average energy per standing wave. According to the **theorem of equipartition of energy**, a mainstay of classical physics, the average energy per degree of freedom of an entity (such as a molecule of an ideal gas) that is a member of a system of such entities in thermal equilibrium at the temperature T is $\frac{1}{2}kT$. Here k is **Boltzmann's constant**:

Boltzmann's constant

$$k = 1.381 \times 10^{-23} \text{ J/K}$$

A degree of freedom is a mode of energy possession. Thus a monatomic ideal gas molecule has three degrees of freedom, corresponding to kinetic energy of motion in three independent directions, for an average total energy of $\frac{3}{2}kT$.

A one-dimensional harmonic oscillator has two degrees of freedom, one that corresponds to its kinetic energy and one that corresponds to its potential energy. Because each standing wave in a cavity originates in an oscillating electric charge in the cavity wall, two degrees of freedom are associated with the wave and it should have an average energy of $2(\frac{1}{2})kT$:

Classical average energy per standing wave

$$\bar{\epsilon} = kT \quad (2.2)$$

The total energy $u(\nu) d\nu$ per unit volume in the cavity in the frequency interval from ν to $\nu + d\nu$ is therefore

Rayleigh-Jeans formula

$$u(\nu) d\nu = \bar{\epsilon} G(\nu) d\nu = \frac{8\pi kT}{c^3} \nu^2 d\nu \quad (2.3)$$

This radiation rate is proportional to this energy density for frequencies between ν and $\nu + d\nu$. Equation (2.3), the **Rayleigh-Jeans formula**, contains everything that classical physics can say about the spectrum of blackbody radiation.

Even a glance at Eq. (2.3) shows that it cannot possibly be correct. As the frequency ν increases toward the ultraviolet end of the spectrum, this formula predicts that the energy density should increase as ν^2 . In the limit of infinitely high frequencies, $u(\nu) d\nu$ therefore should also go to infinity. In reality, of course, the energy density (and radiation rate) falls to 0 as $\nu \rightarrow \infty$ (Fig. 2.8). This discrepancy became known as the **ultraviolet catastrophe** of classical physics. Where did Rayleigh and Jeans go wrong?

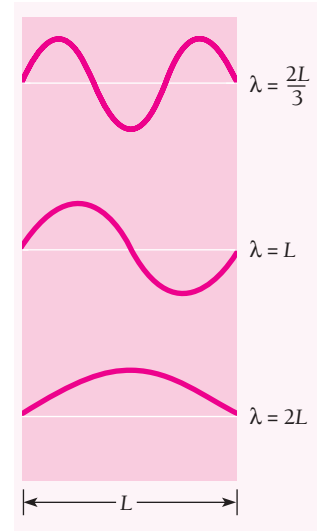


Figure 2.7 Em radiation in a cavity whose walls are perfect reflectors consists of standing waves that have nodes at the walls, which restricts their possible wavelengths. Shown are three possible wavelengths when the distance between opposite walls is L .

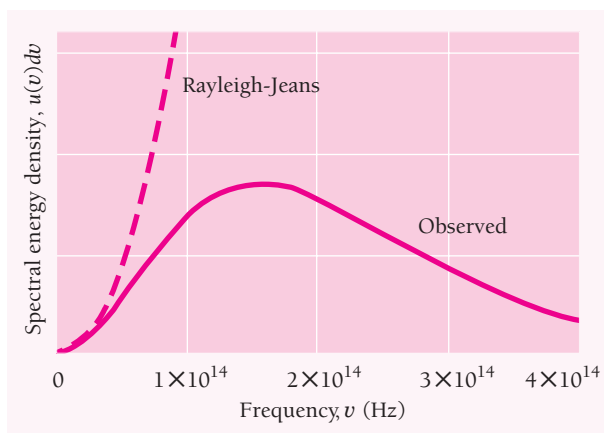


Figure 2.8 Comparison of the Rayleigh-Jeans formula for the spectrum of the radiation from a blackbody at 1500 K with the observed spectrum. The discrepancy is known as the ultraviolet catastrophe because it increases with increasing frequency. This failure of classical physics led Planck to the discovery that radiation is emitted in quanta whose energy is $h\nu$.

Planck Radiation Formula

In 1900 the German physicist Max Planck used “lucky guesswork” (as he later called it) to come up with a formula for the spectral energy density of blackbody radiation:

Planck radiation formula
$$u(\nu) d\nu = \frac{8\pi h}{c^3} \frac{\nu^3 d\nu}{e^{h\nu/kT} - 1} \quad (2.4)$$

Here h is a constant whose value is

Planck’s constant
$$h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s}$$



Max Planck (1858–1947) was born in Kiel and educated in Munich and Berlin. At the University of Berlin he studied under Kirchhoff and Helmholtz, as Hertz had done earlier. Planck realized that blackbody radiation was important because it was a fundamental effect independent of atomic structure, which was still a mystery in the late nineteenth century, and worked at understanding it for six years before finding the formula the radiation obeyed. He “strived from the day of its discovery to give it a real physical interpretation.” The result was the discovery that radiation is emitted in energy steps of $h\nu$. Although this discovery, for which he received the Nobel Prize in 1918, is now considered to mark the start of

modern physics, Planck himself remained skeptical for a long time of the physical reality of quanta. As he later wrote, “My vain attempts to somehow reconcile the elementary quantum with classical theory continued for many years and cost me great effort. . . . Now I know for certain that the quantum of action has a much more fundamental significance than I originally suspected.”

Like many physicists, Planck was a competent musician (he sometimes played with Einstein) and in addition enjoyed mountain climbing. Although Planck remained in Germany during the Hitler era, he protested the Nazi treatment of Jewish scientists and lost his presidency of the Kaiser Wilhelm Institute as a result. In 1945 one of his sons was implicated in a plot to kill Hitler and was executed. After World War II the Institute was renamed after Planck and he was again its head until his death.

At high frequencies, $h\nu \gg kT$ and $e^{h\nu/kT} \rightarrow \infty$, which means that $u(\nu) d\nu \rightarrow 0$ as observed. No more ultraviolet catastrophe. At low frequencies, where the Rayleigh-Jeans formula is a good approximation to the data (see Fig. 2.8), $h\nu \ll kT$ and $h\nu/kT \ll 1$. In general,

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

If x is small, $e^x \approx 1 + x$, and so for $h\nu/kT \ll 1$ we have

$$\frac{1}{e^{h\nu/kT} - 1} \approx \frac{1}{1 + \frac{h\nu}{kT} - 1} \approx \frac{kT}{h\nu} \quad h\nu \ll kT$$

Thus at low frequencies Planck's formula becomes

$$u(\nu) d\nu \approx \frac{8\pi h}{c^3} \nu^3 \left(\frac{kT}{h\nu} \right) d\nu \approx \frac{8\pi kT}{c^3} \nu^2 d\nu$$

which is the Rayleigh-Jeans formula. Planck's formula is clearly at least on the right track; in fact, it has turned out to be completely correct.

Next Planck had the problem of justifying Eq. (2.4) in terms of physical principles. A new principle seemed needed to explain his formula, but what was it? After several weeks of “the most strenuous work of my life,” Planck found the answer: The oscillators in the cavity walls could not have a continuous distribution of possible energies ϵ but must have only the specific energies

$$\text{Oscillator energies} \quad \epsilon_n = nh\nu \quad n = 0, 1, 2, \dots \quad (2.5)$$

An oscillator emits radiation of frequency ν when it drops from one energy state to the next lower one, and it jumps to the next higher state when it absorbs radiation of frequency ν . Each discrete bundle of energy $h\nu$ is called a **quantum** (plural **quanta**) from the Latin for “how much.”

With oscillator energies limited to $nh\nu$, the average energy per oscillator in the cavity walls—and so per standing wave—turned out to be not $\bar{\epsilon} = kT$ as for a continuous distribution of oscillator energies, but instead

$$\text{Actual average energy per standing wave} \quad \epsilon = \frac{h\nu}{e^{h\nu/kT} - 1} \quad (2.6)$$

This average energy leads to Eq. (2.4). Blackbody radiation is further discussed in Chap. 9.

Example 2.1

Assume that a certain 660-Hz tuning fork can be considered as a harmonic oscillator whose vibrational energy is 0.04 J. Compare the energy quanta of this tuning fork with those of an atomic oscillator that emits and absorbs orange light whose frequency is 5.00×10^{14} Hz.

Solution

(a) For the tuning fork,

$$h\nu_1 = (6.63 \times 10^{-34} \text{ J} \cdot \text{s}) (660 \text{ s}^{-1}) = 4.38 \times 10^{-31} \text{ J}$$

The total energy of the vibrating tines of the fork is therefore about 10^{29} times the quantum energy $h\nu$. The quantization of energy in the tuning fork is obviously far too small to be observed, and we are justified in regarding the fork as obeying classical physics.

(b) For the atomic oscillator,

$$h\nu_2 = (6.63 \times 10^{-34} \text{ J} \cdot \text{s}) (5.00 \times 10^{14} \text{ s}^{-1}) = 3.32 \times 10^{-19} \text{ J}$$

In electronvolts, the usual energy unit in atomic physics,

$$h\nu_2 = \frac{3.32 \times 10^{-19} \text{ J}}{1.60 \times 10^{-19} \text{ J/eV}} = 2.08 \text{ eV}$$

This is a significant amount of energy on an atomic scale, and it is not surprising that classical physics fails to account for phenomena on this scale.

The concept that the oscillators in the cavity walls can interchange energy with standing waves in the cavity only in quanta of $h\nu$ is, from the point of view of classical physics, impossible to understand. Planck regarded his quantum hypothesis as an “act of desperation” and, along with other physicists of his time, was unsure of how seriously to regard it as an element of physical reality. For many years he held that, although the energy transfers between electric oscillators and em waves apparently are quantized, em waves themselves behave in an entirely classical way with a continuous range of possible energies.

2.3 PHOTOELECTRIC EFFECT

The energies of electrons liberated by light depend on the frequency of the light

During his experiments on em waves, Hertz noticed that sparks occurred more readily in the air gap of his transmitter when ultraviolet light was directed at one of the metal balls. He did not follow up this observation, but others did. They soon discovered that the cause was electrons emitted when the frequency of the light was sufficiently high. This phenomenon is known as the **photoelectric effect** and the emitted electrons are called **photoelectrons**. It is one of the ironies of history that the same work to demonstrate that light consists of em waves also gave the first hint that this was not the whole story.

Figure 2.9 shows how the photoelectric effect was studied. An evacuated tube contains two electrodes connected to a source of variable voltage, with the metal plate whose surface is irradiated as the anode. Some of the photoelectrons that emerge from this surface have enough energy to reach the cathode despite its negative polarity, and they constitute the measured current. The slower photoelectrons are repelled before they get to the cathode. When the voltage is increased to a certain value V_0 , of the order of several volts, no more photoelectrons arrive, as indicated by the current dropping to zero. This extinction voltage corresponds to the maximum photoelectron kinetic energy.