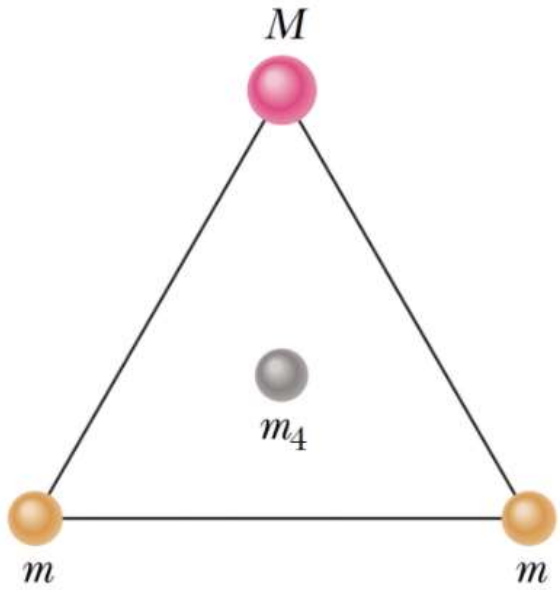
GP1-L12

Problem 1

As seen in the figure, two spheres of mass m and a third sphere of mass M form an equilateral triangle, and a fourth sphere of mass m4 is at the center of the triangle. The net gravitational force on that central sphere from the three other spheres is zero. (a) What is M in terms of m? (b) If we double the value of m4, what then is the magnitude of the net gravitational force on the central sphere? (02小題)



$$\vec{F} = G\frac{m_1 m_2}{r^2}\hat{\mathrm{r}}.$$

(a)M=___ m

$$r = \ell/2\cos 30^\circ = \ell/\sqrt{3}$$

$$3\frac{Gm_4 m}{\ell^2} = \frac{Gm_4 M}{\left(\ell/\sqrt{3}\right)^2}$$

$$2F_y = 2\left(\frac{Gm_4 m}{r^2}\right)\sin 30^\circ = 3\frac{Gm_4 m}{\ell^2}.$$

$$m = M.$$

Problem 2

(a) What will an object weigh on the Moon's surface if it weighs 100 N on Earth's surface? (b) How many Earth radii must this same object be from the center of Earth if it is to weigh the same as it does on the Moon?

(a)如果一個物體在地球表面的重量為100 N，它在月球表面上重量將是？ (b)如果同一物體受地球的重力與在月球的重量相同，那麼該物體必須距地球中心多少個地球半徑？ (02小題)

(a)W=____ N

$g_{\text{moon}} = 1.67\ \text{m/s}^2$

03: **ANS:=17**

(b)how many radii? r= _____ R_E

$$W_{\text{moon}} = (100\ \text{N})(1.67/9.8) = 17\ \text{N}.$$

04: **ANS:=2.4**

$$a_g = 1.67\ \text{m/s}^2.$$

$$a_g = \frac{Gm_E}{r^2} \Rightarrow r = \sqrt{\frac{Gm_E}{a_g}} = 1.5 \times 10^7\ \text{m} \qquad \frac{1.5 \times 10^7}{6.38 \times 10^6} = 2.35$$

$$r/R_E = 2.4$$

Problem 3

Left over from the big-bang beginning of the universe, tiny black holes might still wander through the universe. If one with a mass of 1×10^{11} kg (and a radius of only 1×10^{-16} m reached Earth, at what distance from your head would its gravitational pull on you match that of Earth's? Treat your body as a point particle.

從宇宙大爆炸的開始遺留下來的，微小的黑洞可能仍然在宇宙中徘徊。 如果一個質量為1×10^{11} kg（半徑僅為1×10^{-16} m的黑洞到達地球，它的引力在距您的頭部多少距離的地方拉你的力量與地球的重力相等？將你的身體視為一個質點。 (01小題)

$$F_g = \frac{GM_E m}{R_E^2} = mg \qquad mg = \frac{GM_E m}{R_E^2} = \frac{GM_b m}{r^2} \Rightarrow$$

$$g = GM_E / R_E^2 = 9.8\ \text{m/s}^2 \qquad r = \sqrt{\frac{GM_b}{g}}$$

$$M_b = 1 \times 10^{11}\ \text{kg}$$

$$F_g = \frac{GM_b m}{r^2} = mg \qquad = \sqrt{\frac{(6.67 \times 10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(1 \times 10^{11}\ \text{kg})}{9.8\ \text{m/s}^2}} \approx 0.8\ \text{m}.$$

Problem 4

One model for a certain planet has a core of radius R and mass M surrounded by an outer shell of inner radius R, outer radius 2R, and mass 4M. If $M = 4.1 \times 10^{24}$ kg and $R = 6.0 \times 10^{6}$ m, what is the gravitational acceleration of a particle at points (a) R and (b) 3R from the center of the planet?

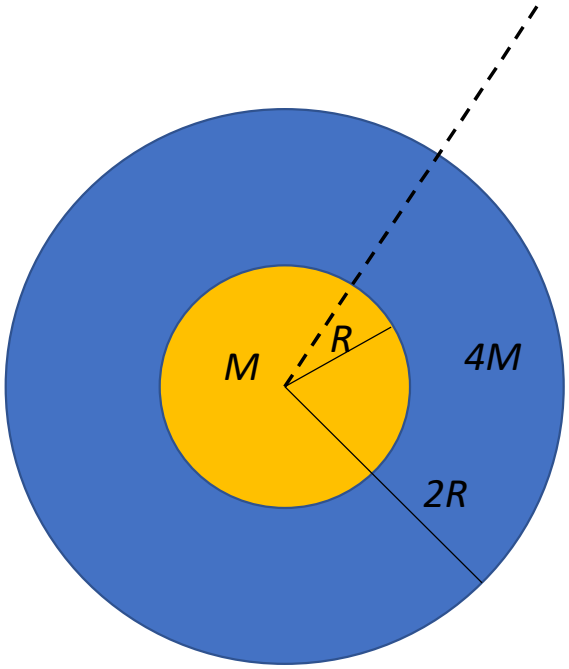
某個行星的一個模型具有一個半徑R和質量M的核，並被一個內部半徑R，外部半徑2R和質量4M的外殼包圍。 如果$M = 4.1 \times 10^{24}$ kg和$R = 6.0 \times 10^{6}$ m，則粒子在距離行星中心(a)R和(b)3R點處的重力加速度是多少？ (02小題)

(a)a_g=____ m/s^2

06: **ANS:=7.6**

(b)a_g=____ m/s^2

07: **ANS:=4.2**



$$a_g = \frac{GM}{R^2} = 7.6 \text{ m/s}^2.$$

$$a_g = \frac{G(5M)}{(3R)^2} = 4.2 \text{ m/s}^2.$$

Problem 5

Certain neutron stars (extremely dense stars) are believed to be rotating at about 1 rev/s. If such a star has a radius of 20 km, what must be its minimum mass so that material on its surface remains in place during the rapid rotation?

據信某些中子星（極緻密星）的自轉速度約為1轉/秒。 如果此類星體的半徑為20 km，那麼它的最小質量必須是多少，以便在快速旋轉過程中其表面上的物質可停留在表位？ (01小題)

"g" becomes zero,

$$0 = \frac{GM}{R^2} - R\omega^2 \Rightarrow M = \frac{R^3\omega^2}{G}.$$

$$R = 20000 \text{ m and } \omega = 2\pi \text{ rad/s},$$

$$M = 4.7 \times 10^{24} \text{ kg} \approx 5 \times 10^{24} \text{ kg}.$$

Neutron Stars

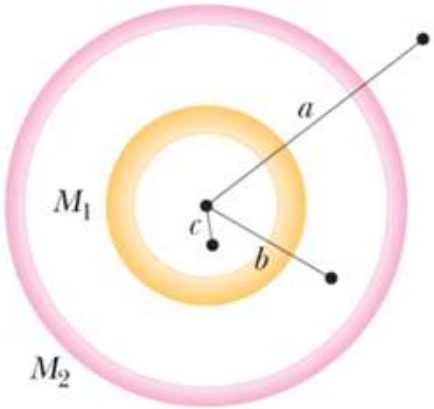
A neutron star is about 20 km in diameter and has the mass of about 1.4 times that of our Sun. This means that a neutron star is so dense that on Earth, one teaspoonful would weigh a billion tons! Because of its small size and high density, a neutron star possesses a surface gravitational field about 2 x 10^{11} times that of Earth. Neutron stars can also have magnetic fields a million times stronger than the strongest magnetic fields produced on Earth.

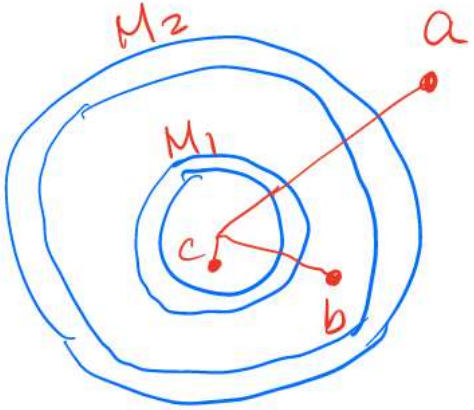
https://imagine.gsfc.nasa.gov/science/objects/pulsars1.html.old

Problem 6

Two concentric spherical shells with uniformly distributed masses M_1=M_1 and M_2=M_2 are situated as shown in the figure. Find the magnitude of the net gravitational force on a particle of mass m, due to the shells, when the particle is located at radial distance (a) a, (b) b, and (c) c.

如圖所示，放置了兩個質量均勻分佈的同心球殼M_1= M_1和M_2= M_2。 當質點位於徑向距離(a)a，(b)b和(c)c時，由於殼的作用，求出質量為m的質點上的淨重力的大小。 (03小題)





a在整個球体外

$$F_a = \frac{G(M_1+M_2)M}{a^2}$$

b所受的重力只與M_1有关

$$F_b = \frac{G M_1 m}{b^2}$$

c在M_1, M_2的內部

$$F_c = 0$$

(a)$r = a, F_g =$_____ [M_1,M_2,m,a,b,c,G]

09: **ANS:=(G*(M_1+M_2)*m)/a**2**

(a)$r = b, F_g =$_____ [M_1,M_2,m,a,b,c,G]

10: **ANS:=(G*M_1*m)/b**2**

(a)$r = c, F_g =$_____ [M_1,M_2,m,a,b,c,G]

11: **ANS:=0**

Problem 7

The mean diameters of Mars and Earth are 6.9×10^3 km and 1.3×10^4 km, respectively. The mass of Mars is 0.11 times Earth's mass. (a) What is the ratio of the mean density (mass per unit volume) of Mars to that of Earth? (b) What is the value of the gravitational acceleration on Mars? (c) What is the escape speed on Mars?

火星和地球的平均直徑分別為6.9×10^3 km和1.3×10^4 km。 火星的質量是地球質量的0.11倍。 (a)火星與地球的平均密度（單位體積質量）之比是多少？ (b)火星上的重力加速度值是多少？ (c)火星上的逃逸速度是多少？ (03小題)

$$\rho = 3M/4\pi R^3,$$

$$\frac{\rho_M}{\rho_E} = \frac{M_M}{M_E}\frac{R_E^3}{R_M^3} = 0.11\left(\frac{0.65\times10^4\ \text{km}}{3.45\times10^3\ \text{km}}\right)^3 = 0.74.$$

$$\frac{1}{2}mv^2 = G\frac{mM}{R} \quad \Rightarrow \quad v = \sqrt{\frac{2GM}{R}}.$$

$$a_g = GM/R^2,$$

$$v = \sqrt{\frac{2(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(0.11)(5.98\times10^{24}\ \text{kg})}{3.45\times10^6\ \text{m}}} = 5.0\times10^3\ \text{m/s}.$$

$$a_{gM} = \frac{M_M}{M_E}\frac{R_E^2}{R_M^2}a_{gE} = 0.11\left(\frac{0.65\times10^4\ \text{km}}{3.45\times10^3\ \text{km}}\right)^2(9.8\ \text{m/s}^2) = 3.8\ \text{m/s}^2.$$

(a) $\frac{\rho_{Mars}}{\rho_{Earth}}$ =_____

12: **ANS:=0.74**

(b) g_{Mars} =_____ m/s^2

13: **ANS:=3.8**

(c) $v_{\text{esc,Mars}}$ =_____ m/s

14: **ANS:=5000**

Problem 8

A projectile is shot directly away from Earth's surface. Neglect the rotation of Earth. What multiple of Earth's radius R_E gives the radial distance a projectile reaches if (a) its initial speed is 0.500 of the escape speed from Earth and (b) its initial kinetic energy is 0.500 of the kinetic energy required to escape Earth?

一拋體直接從地球表面射出。忽略地球的自轉。如果(a)初始速度是從地球脫離速度的0.5，則拋體到達的徑向距離是地球半徑R_E的倍數。（b）初始動能是脫離地球所需動能的0.5，則拋體到達的徑向距離是地球半徑R_E的倍數。(02小題)

(a) From Eq. 13-28, we see that $v_0 = \sqrt{GM/2R_E}$ in this problem. Using energy conservation, we have

$$\frac{1}{2}mv_0^2 - GMm/R_E = -GMm/r$$

which yields $r = 4R_E/3$. So the multiple of R_E is 4/3 or 1.33.

(a)the distance it reaches, r=____ R_E

15: **ANS:=1.333**

(b)the distance it reaches, r=____ R_E

16: **ANS:=2**

(b) Using the equation in the textbook immediately preceding Eq. 13-28, we see that in this problem we have $K_i = GMm/2R_E$, and the above manipulation (using energy conservation) in this case leads to $r = 2R_E$. So the multiple of R_E is 2.00.

Problem 9

The Martian satellite Phobos travels in an approximately circular orbit of radius 9.4×10^6 m with a period of 7 h 39 min. Calculate the mass of Mars from this information.

火星衛星火衛一在半徑為9.4×10^6 m的近似圓形軌道中移動，週期為7 h 39 min。 根據這些信息計算火星的質量。

(01小題)

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mass of Mars=_____ kg

17: **ANS:=6.5E23**

$$T^2 = \frac{4\pi^2}{GM} r^3$$

$$M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2 (9.4 \times 10^6)^3}{(6.67 \times 10^{-11})(2.754 \times 10^4)^2} = 6.5 \times 10^{23}$$

Problem 10

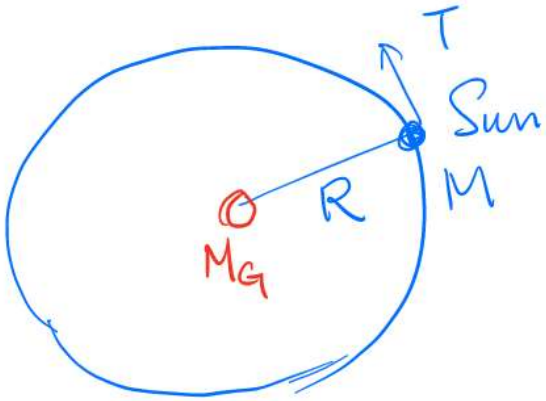
The Sun, which is 2.2×10^{20} m from the center of the Milky Way galaxy, revolves around that center once every 2.5×10^{8} years. Assuming each star in the Galaxy has a mass equal to the Sun's mass of 2.0×10^{30} kg, the stars are distributed uniformly in a sphere about the galactic center, and the Sun is at the edge of that sphere, estimate the number of stars in the Galaxy.

太陽距銀河系中心2.2×10^{20} m，每2.5×10^{8}年圍繞該中心旋轉一次。 假設銀河系中的每顆恆星的質量等於太陽的質量2.0×10^{30} kg，這些恆星均勻分佈在銀河系中心周圍的一個球體中，並且太陽位於該恆星球體的邊緣，請估計銀河系中的恆星數。 (01小題)

the number of stars in the Galaxy=_____

18: **ANS:=5.1E10**

銀河星系恆星數量
1000–4000億
2.5×10^{11}



$$M_G = \frac{4\pi^2 R^3}{GT^2}$$

$$N = \frac{M_G}{M} = \frac{4\pi^2 R^3}{GT^2 M}$$

$$= \frac{4\pi^2 (2.2\times10^{20})^3}{6.67\times10^{-11}(2.5\times10^{8}\times365\times86400)^2\times2\times10^{30}}$$

$$= 5.07\times10^{-15}\times10^{(60+11-16-30)}$$

$$= 5.07\times10^{10}$$