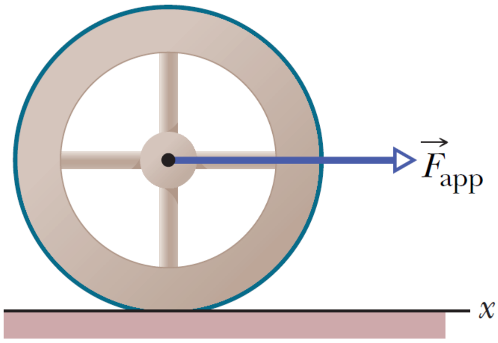
GP1-L11

Problem 1

In the figure, constant horizontal force of magnitude 10 N is applied to a wheel of mass 10 kg and radius 0.30 m. The wheel rolls smoothly on the horizontal surface, and the acceleration of its center of mass has magnitude 0.60 m/s^2. (a)What is the frictional force on the wheel? (b) What is the rotational inertia of the wheel about the rotation axis through its center of mass? (02小題)



(a)magnitude of the frictional force on the wheel=_____ N

01: **ANS:=4**

(b)the rotational inertia of the wheel=______ kg.m^2

02: **ANS:=0.6**

Solution:

$$F_{app} - f_s = ma \quad \Rightarrow f_s = 10\text{N} - (10\text{kg})(0.60\ \text{m/s}^2) = 4.0\ \text{N}.$$

$$\vec{f}_s = (-4.0\ \text{N})\hat{\text{i}}$$

the acceleration of its center of mass has magnitude 0.60 m/s^2.

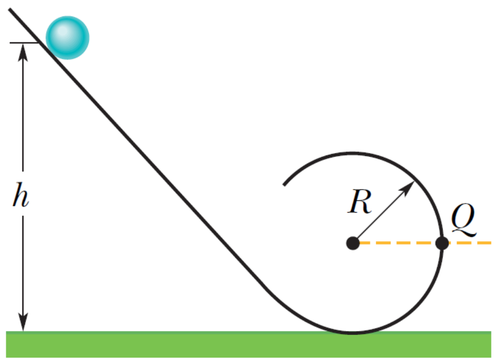
$$|\alpha| = |a_{com}| / R = 2.0\ \text{rad/s}^2, \qquad R = 0.30\ \text{m},$$

The only force not directed towards (or away from) the center of mass is $\vec{f}_s$,

$$|\tau| = I|\alpha| \quad \Rightarrow \quad (0.30\,\text{m})(4.0\,\text{N}) = I(2.0\,\text{rad/s}^2)$$

Problem 2

In the figure, a solid brass ball of mass 0.280 g will roll smoothly along a loop-the-loop track when released from rest along the straight section. The circular loop has radius R = 14.0 cm, and the ball has radius $r << R$. (a) What is h if the ball is on the verge of leaving the track when it reaches the top of the loop? If the ball is released at height h = 6 R, what are the (b) magnitude of the horizontal force component acting on the ball at point Q? (02小題)



(a)h=_____ cm

[03:](#) **ANS:=<u>37.8</u>**

(b) N=______ N

[04:](#) **ANS:=<u>1.96E-2</u>**

Solution:

8. Using the floor as the reference position for computing potential energy, mechanical energy conservation leads to

$$U_{\text{release}} = K_{\text{top}} + U_{\text{top}} \quad \Rightarrow \quad mgh = \frac{1}{2}mv_{\text{com}}^2 + \frac{1}{2}I\omega^2 + mg(2R).$$

Substituting $I = \frac{2}{5}mr^2$ (Table 10-2(f)) and $\omega = v_{\text{com}}/r$ (Eq. 11-2), we obtain

$$mgh = \frac{1}{2}mv_{\text{com}}^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v_{\text{com}}}{r}\right)^2 + 2mgR \quad \Rightarrow \quad gh = \frac{7}{10}v_{\text{com}}^2 + 2gR$$

where we have canceled out mass m in that last step.

(a) To be on the verge of losing contact with the loop (at the top) means the normal force is vanishingly small. In this case, Newton's second law along the vertical direction (+y downward) leads to

$$mg = ma_r \Rightarrow g = \frac{v_{\text{com}}^2}{R-r}$$

where we have used Eq. 10-23 for the radial (centripetal) acceleration (of the center of mass, which at this moment is a distance $R - r$ from the center of the loop). Plugging the result $v_{\text{com}}^2 = g(R-r)$ into the previous expression stemming from energy considerations gives

$$gh = \frac{7}{10}(g)(R-r) + 2gR$$

which leads to $h = 2.7R - 0.7r \approx 2.7R$. With R = 14.0 cm , we have h = (2.7)(14.0 cm) = 37.8 cm.

(b) The energy considerations shown above (now with h = 6R) can be applied to point Q (which, however, is only at a height of R) yielding the condition

$$g(6R) = \frac{7}{10}v_{\text{com}}^2 + gR$$

which gives us $v_{\text{com}}^2 = 50gR/7$. Recalling previous remarks about the radial acceleration, Newton's second law applied to the horizontal axis at Q leads to

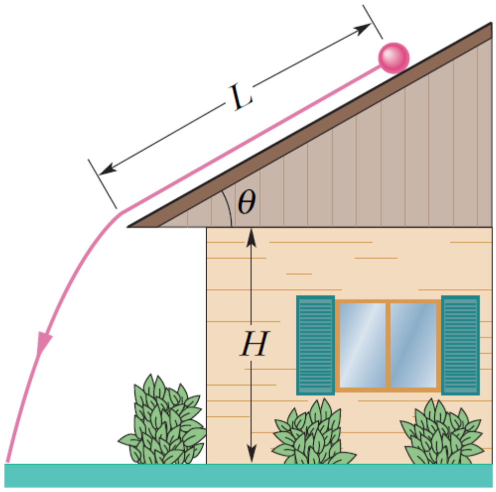
$$N = m\frac{v_{\text{com}}^2}{R-r} = m\frac{50gR}{7(R-r)}$$

which (for $R \gg r$) gives

$$N \approx \frac{50mg}{7} = \frac{50(2.80\times10^{-4}\text{ kg})(9.80\text{ m/s}^2)}{7} = 1.96\times10^{-2}\text{ N}.$$

Problem 3

In the figure, a solid cylinder of radius 10 cm and mass 12 kg starts from rest and rolls without slipping a distance L = 6.0 m down a roof that is inclined at the angle $\theta = 30^{\circ}$. (a)What is the angular speed of the cylinder about its center as it leaves the roof? (b) The roof's edge is at height H = 5.0 m. How far horizontally from the roof's edge does the cylinder hit the level ground? (02小題)



(a)the angular speed, ω=____ rad./s

05: **ANS:=63**

(b)the distance from the roof's edge=_____ m

06: **ANS:=4**

Solution:

$h = 6.0 \sin 30° = 3.0 \text{ m}$

$U_i = Mgh$

$K_f = \tfrac{1}{2} Mv^2 + \tfrac{1}{2} I\omega^2 .$

$v = R\omega \qquad I = \tfrac{1}{2} MR^2$

$$Mgh = \frac{1}{2} Mv^2 + \frac{1}{2} I\omega^2 = \frac{1}{2} MR^2\omega^2 + \frac{1}{4} MR^2\omega^2 = \frac{3}{4} MR^2\omega^2 .$$

$$\omega = \frac{1}{R}\sqrt{\frac{4}{3} gh} = \frac{1}{0.10 \text{ m}}\sqrt{\frac{4}{3}(9.8 \text{ m/s}^2)(3.0 \text{ m})} = 63 \text{ rad/s}.$$

$v_0 = R\omega = 6.3$ m/s,

$v_{0x} = v_0 \cos 30° = 5.4 \text{ m/s}$

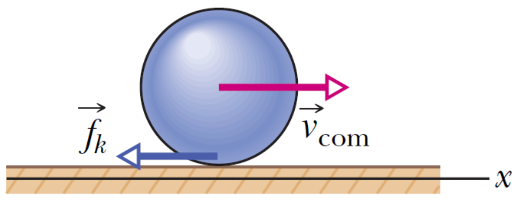
$v_{0y} = v_0 \sin 30° = 3.1 \text{ m/s}.$

$x = v_{0x} t$ and $y = v_{0y} t + \dfrac{1}{2} gt^2 .$

$y = H = 5.0 \text{ m} \qquad t = \dfrac{-v_{0y} + \sqrt{v_{0y}^2 + 2gH}}{g} = 0.74 \text{ s}.$

$x = (5.4 \text{ m/s})(0.74 \text{ s}) = 4.0 \text{ m}.$

Problem 4



In the figure, a bowler throws a bowling ball of radius R = 11 cm along a lane. The ball slides on the lane with initial speed $v_{com,0} = 8.5$ m/s and initial angular speed $\omega_0 = 0$. The coefficient of kinetic friction between the ball and the lane is 0.21. The kinetic frictional force acting on the ball causes a linear acceleration of the ball while producing a torque that causes an angular acceleration of the ball. When speed v_{com} has decreased enough and angular speed ω has increased enough, the ball stops sliding and then rolls smoothly. v_{com} can be expressed in terms of ω: $v_{com} = R\omega$. During the sliding, what are the ball's (a) linear acceleration and (b) angular acceleration? (c) How long does the ball slide? (d) How far does the ball slide? (e) What is the linear speed of the ball when smooth rolling begins? Notice the sign of the linear displacment and angular displacement: we define positive signs for moving in the +x (right) and rotation in counterclockwise. You have to indicate the sign in your answers. (05小題)

(a)linear acceleration a=______ m/s^2

07: **ANS:=-2.1**

(b)angular acceleration α=______ rad/s^2

08: **ANS:=-47**

(c)the time period of sliding=_____ s

09: **ANS:=1.2**

(d)the sliding distance=_____ m

10: **ANS:=8.6**

(e)the linear speed of the ball when smooth rolling begins, v=_____ m/s

11: **ANS:=6.1**

Solution:

$$v_{com} = -R\omega = (-0.11 \text{ m})\omega.$$

force of friction exerted on the ball of mass m is $-\mu_k mg$

$$a_{com} = -\mu g = -(0.21)\,(9.8 \text{ m/s}^2) = -2.1 \text{ m/s}^2$$

$$\tau = -\mu mgR\,.$$

$$\alpha = \frac{\tau}{I} = \frac{-\mu mgR}{2mR^2/5} = \frac{-5\mu g}{2R} = \frac{-5(0.21)(9.8 \text{ m/s}^2)}{2(0.11 \text{ m})} = -47 \text{ rad/s}^2$$

The center-of-mass of the sliding ball decelerates from $v_{com,0}$ to v_{com}

$$v_{com} = v_{com,0} - \mu g t.$$

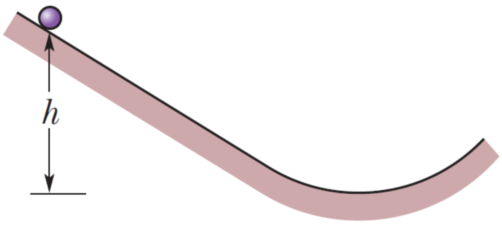
$$|\omega| = |\alpha|\,t = \frac{5\mu g t}{2R} = \frac{v_{com}}{R}$$

$$t = \frac{2v_{com,0}}{7\mu g} = \frac{2(8.5 \text{ m/s})}{7(0.21)(9.8 \text{ m/s}^2)} = 1.2 \text{ s}.$$

$$\Delta x = v_{com,0}t - \frac{1}{2}(\mu g)t^2 = (8.5 \text{ m/s})(1.2 \text{ s}) - \frac{1}{2}(0.21)(9.8 \text{ m/s}^2)(1.2 \text{ s})^2 = 8.6 \text{ m}.$$

$$v_{com} = v_{com,0} - \mu g t = 8.5 \text{ m/s} - (0.21)(9.8 \text{ m/s}^2)(1.2 \text{ s}) = 6.1 \text{ m/s}.$$

Problem 5



In the figure, rolls smoothly from rest down a ramp and onto a circular loop of radius 0.48 m. The initial height of the ball is h = 0.36 m. At the loop bottom, the magnitude of the normal force on the ball is 2Mg. The ball consists of an outer spherical shell (of a certain uniform density) that is glued to a central sphere (of a different uniform density). The rotational inertia of the ball can be expressed in the general form $I = \beta M R^2$, but β is not 0.4 as it is for a ball of uniform density. Determine β. (01小題)

β=_____

12: **ANS:=0.5**

Solution:

$$F_N - Mg = Mv^2/r \qquad F_N = 2Mg)$$

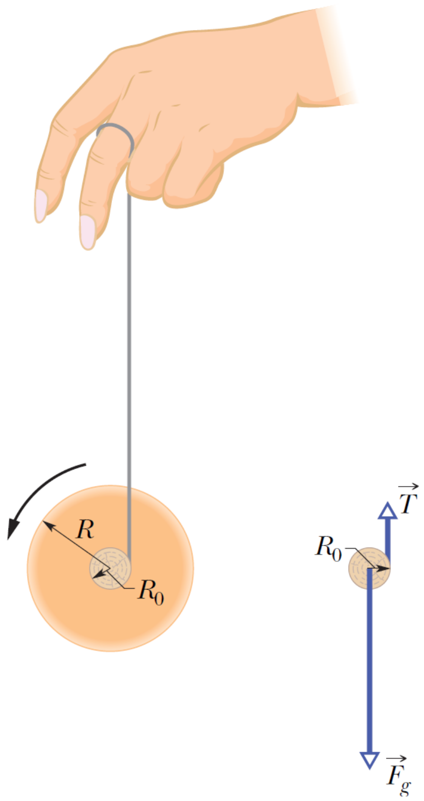
$$\omega^2 = v^2/R^2 = gr/R^2,$$

$$I_{com} = 2MhR^2/r - MR^2 = MR^2[2(0.36/0.48) - 1] .$$

$$\beta = 2(0.36/0.48) - 1 = 0.50.$$

Problem 6

In the figure, A yo-yo has a rotational inertia of 950 g.cm2 and a mass of 120 g. Its axle radius is 3.2 mm, and its string is 120 cm long. The yo-yo rolls from rest down to the end of the string. (a) What is the magnitude of its linear acceleration? (b) How long does it take to reach the end of the string? As it reaches the end of the string, what are its (c) linear speed, (d) translational kinetic energy, (e) rotational kinetic energy, and (f) angular speed? (06小題)



(a)a=____ m/s2

13: **ANS:=12.5**

(b)t=____ s

14: **ANS:=4.38**

(c)|v|=____ m/s

15: **ANS:=0.55**

(d)K_{tr}=_____ J

16: **ANS:=0.018**

(e)K_{rot}=_____ J

17: **ANS:=1.4**

(f)ω=_____ rad/s

18: **ANS:=170**

Solution:

$$a_{\text{com}} = -\frac{g}{1 + I_{\text{com}} / MR_0^2}$$

$$|a_{\text{com}}| = \frac{980 \text{ cm/s}^2}{1 + \left(950 \text{ g} \cdot \text{cm}^2\right) / (120 \text{ g})(0.32 \text{ cm})^2} = 12.5 \text{ cm/s}^2 \approx 13 \text{ cm/s}^2 .$$

$$t = \sqrt{\frac{2y_{\text{com}}}{a_{\text{com}}}} = \sqrt{\frac{2(-120 \text{ cm})}{-12.5 \text{ cm/s}^2}} = 4.38 \text{ s} \approx 4.4 \text{ s}.$$

$$v_{\text{com}} = a_{\text{com}} t = \left(-12.5 \text{ cm/s}^2\right) (4.38 \text{ s}) = -54.8 \text{ cm/s},$$

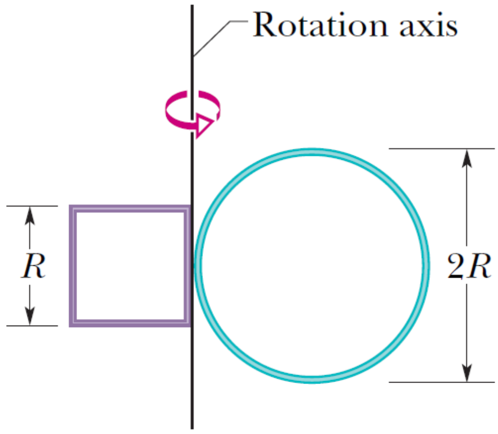
$$\tfrac{1}{2} m v_{\text{com}}^2 = \tfrac{1}{2} (0.120 \text{ kg})(0.548 \text{ m/s})^2 = 1.8 \times 10^{-2} \text{ J} .$$

$$\frac{1}{2} I_{\text{com}} \omega^2 = \frac{1}{2} I_{\text{com}} \frac{v_{\text{com}}^2}{R_0^2} = \frac{1}{2} \frac{\left(9.50 \times 10^{-5} \text{ kg} \cdot \text{m}^2\right)(0.548 \text{ m/s})^2}{\left(3.2 \times 10^{-3} \text{ m}\right)^2}$$

$$K_{\text{rot}} = 1.4 \text{ J}.$$

Problem 7

The figure shows a rigid structure consisting of a circular hoop of radius R and mass m, and a square made of four thin bars, each of length R and mass m. The rigid structure rotates at a constant speed about a vertical axis, with a period of rotation of 2.5 s. Assuming R=0.50 m and m = 2.0 kg, calculate (a) the structure's rotational inertia about the axis of rotation and (b) its angular momentum about that axis. (02小題)



(a)I=____ kg.m^2

19: **ANS:=16**

(b)angular momentum, L=_____ kg.m^2/s

20: **ANS:=4.0**

Solution:

For the hoop,

Of the thin bars (in the form of a square), the member along the rotation axis has no rotational inertia about that axis

$$I_1 = I_{\text{com}} + mh^2 = \frac{1}{2}mR^2 + mR^2 = \frac{3}{2}mR^2 .$$

$$I_2 = I_{\text{com}} + mh^2 = 0 + mR^2 = mR^2 .$$

$$I_3 = I_{\text{com}} + mh^2 = \frac{1}{12}mR^2 + m\left(\frac{R}{2}\right)^2 = \frac{1}{3}mR^2$$

$I_3 = I_4$,

the total rotational inertia is

$$I_1 + I_2 + I_3 + I_4 = \frac{19}{6}mR^2 = 1.6\,\text{kg}\cdot\text{m}^2$$

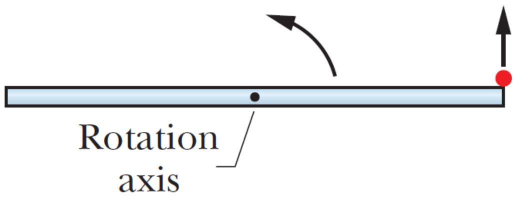
(b) The angular speed is constant:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{2.5} = 2.5\,\text{rad/s}.$$

Thus, $L = I_{\text{total}}\omega = 4.0\,\text{kg}\cdot\text{m}^2/\text{s}.$

Problem 8

The figure is an overhead view of a thin uniform rod of length 0.800 m and mass M rotating horizontally at angular speed 20.0 rad/s about an axis through its center. A particle of mass M/3 initially attached to one end is ejected from the rod and travels along a path that is perpendicular to the rod at the instant of ejection. If the particle's speed v_p is 6 m/s greater than the speed of the rod end just after ejection, what is the value of v_p? (01 小題)



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v_p=_____ m/s

21: **ANS:=<u>11</u>**

Solution:

By angular momentum conservation

$$L_p' + L_r' = L_p + L_r$$

$$\left(\frac{L}{2}\right) m v_p + \frac{1}{12} M L^2 \omega' = I_p \omega + \frac{1}{12} M L^2 \omega$$

$$I_p = m(L/2)^2$$

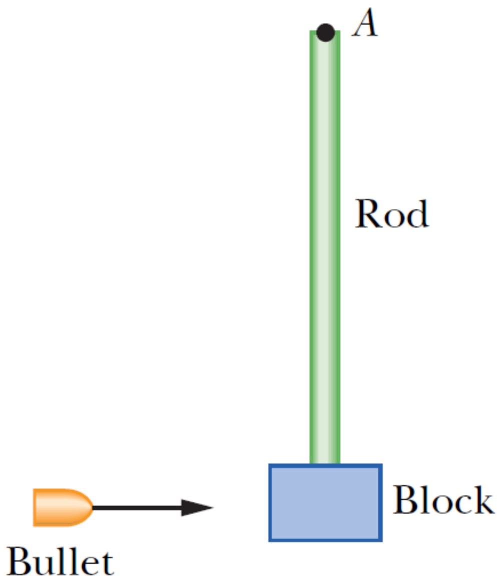
$$\omega' = v_{end}/r = (v_p - 6)/(L/2),$$

$$M = 3m \text{ and } \omega = 20 \text{ rad/s},$$

$$v_p = 11.0 \text{ m/s}.$$

Problem 9

In the figure, a 1.0 g bullet is fired into a 0.50 kg block attached to the end of a 0.60 m nonuniform rod of mass 0.50 kg. The block-rod-bullet system then rotates in the plane of the figure, about a fixed axis at A. The rotational inertia of the rod alone about that axis at A is 0.060 kg.m^2. Treat the block as a particle. (a) What then is the rotational inertia of the block-rod-bullet system about point A? (b) If the angular speed of the system about A just after impact is 4.5 rad/s, what is the bullet's speed just before impact? (02小題)



(a)I=____ kg.m^2

22: **ANS:=0.24**

(b)bullet's speed just before impact, v_b=____ m/s

23: **ANS:=1800**

Solution:

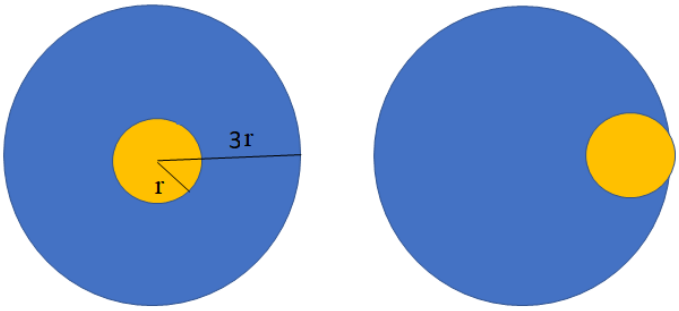
(a) With $r = 0.60$ m, we obtain $I = 0.060 + (0.501)r^2 = 0.24\ \text{kg}\cdot\text{m}^2$.

(b) Invoking angular momentum conservation, with SI units understood,

$$\ell_0 = L_f \;\Rightarrow\; mv_0 r = I\omega \;\Rightarrow\; (0.001)v_0(0.60) = (0.24)(4.5)$$

which leads to $v_0 = 1.8 \times 10^3$ m/s.

Problem 10



A uniform disk of mass 10m and radius 3r can rotate freely about its fixed center like a merry-go-round. A smaller uniform disk of mass m and radius r lies on top of the larger disk, concentric with it. Initially the two disks rotate together with an angular velocity of 20 rad/s. Then a slight disturbance causes the smaller disk to slide outward across the larger disk, until the outer edge of the smaller disk catches on the outer edge of the larger disk. Afterward, the two disks again rotate together (without further sliding). (a) What then is their angular velocity about the center of the larger disk? (b) What is the ratio K/K_0 of the new kinetic energy of the two-disk system to the system's initial kinetic energy? (02小題)

(a)ω=_____ rad/s

24: **ANS:=18**

(b)the ratio K/K_0=_____

25: **ANS:=0.919**

Solution: