# GP1-L10

**Problem 1**
The angular position of a point on the rim of a rotating wheel is given by $\theta(t) = 4.0t - 3.0t^2 + t^3$, where $\theta$ is in radians and t is in seconds. What are the angular velocities at (a) t = 2.0 s and (b) t = 4.0 s? (c) What is the average angular acceleration for the time interval that begins at t = 2.0 s and ends at t = 4.0 s? What are the instantaneous angular accelerations at (d) the beginning and (e) the end of this time interval? (05小題)

(a)$\omega(2)$=_______ rad./s

01: **ANS:=4.0**

(b)$\omega(4)$=_______ rad./s

02: **ANS:=28**

(c)the average angular acceleration=______ rad./s

03: **ANS:=12**

(d)$\alpha(2)$=______ rad./s$^2$

04: **ANS:=6**

(e)$\alpha(4)$=______ rad./s$^2$

05: **ANS:=18**

# Solution:

If we make the units explicit, the function is

$$\theta = (4.0 \text{ rad / s})t - (3.0 \text{ rad / s}^2)t^2 + (1.0 \text{ rad / s}^3)t^3$$

but generally we will proceed as shown in the problem—letting these units be understood. Also, in our manipulations we will generally not display the coefficients with their proper number of significant figures.

(a) Eq. 10-6 leads to

$$\omega = \frac{d}{dt}(4t - 3t^2 + t^3) = 4 - 6t + 3t^2.$$

Evaluating this at $t = 2$ s yields $\omega_2 = 4.0$ rad/s.

(b) Evaluating the expression in part (a) at $t = 4$ s gives $\omega_4 = 28$ rad/s.

(c) Consequently, Eq. 10-7 gives

$$\alpha_{\text{avg}} = \frac{\omega_4 - \omega_2}{4 - 2} = 12 \text{ rad / s}^2.$$

(d) And Eq. 10-8 gives

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}(4 - 6t + 3t^2) = -6 + 6t.$$

Evaluating this at $t = 2$ s produces $\alpha_2 = 6.0$ rad/s$^2$.

(e) Evaluating the expression in part (d) at $t = 4$ s yields $\alpha_4 = 18$ rad/s$^2$. We note that our answer for $\alpha_{\text{avg}}$ does turn out to be the arithmetic average of $\alpha_2$ and $\alpha_4$ but point out that this will not always be the case.

**Problem 2**

**A.**The angular speed of an automobile engine is increased at a constant rate from 1200 rev/min to 3000 rev/min in 12 s. (a) What is its angular acceleration? (b) How many revolutions does the engine make during this 12 s interval? (02小題)

(a)$\alpha$=_____ rad./s$^2$

06: **ANS:=15.708**

(b)number of revolutions=______

07: **ANS:=420**

# Solution:

(a) We assume the sense of rotation is positive. Applying Eq. 10-12, we obtain

$$\omega = \omega_0 + \alpha t \quad \Rightarrow \quad \alpha = \frac{(3000 - 1200)\ \text{rev/min}}{(12/60)\ \text{min}} = 9.0\times10^3\ \text{rev/min}^2.$$

(b) And Eq. 10-15 gives

$$\theta = \frac{1}{2}(\omega_0 + \omega)t = \frac{1}{2}(1200\ \text{rev/min} + 3000\ \text{rev/min})\left(\frac{12}{60}\ \text{min}\right) = 4.2\times10^2\ \text{rev}.$$

**Problem 2**
**B.**What are the magnitudes of (a) the angular velocity, (b) the radial acceleration, and (c) the tangential acceleration of a spaceship taking a circular turn of radius 3220 km at a constant speed of 29000 km/h? (03小題)

(a)$\omega$=_____ rad./s

08: **ANS:=2.5E-3**

(b)radial acceleration, $a_r$=______ m/s$^2$

09: **ANS:=20.2**

(c)tangential acceleration, $a_t$=______ m/s$^2$

10: **ANS:=0**

# Solution:

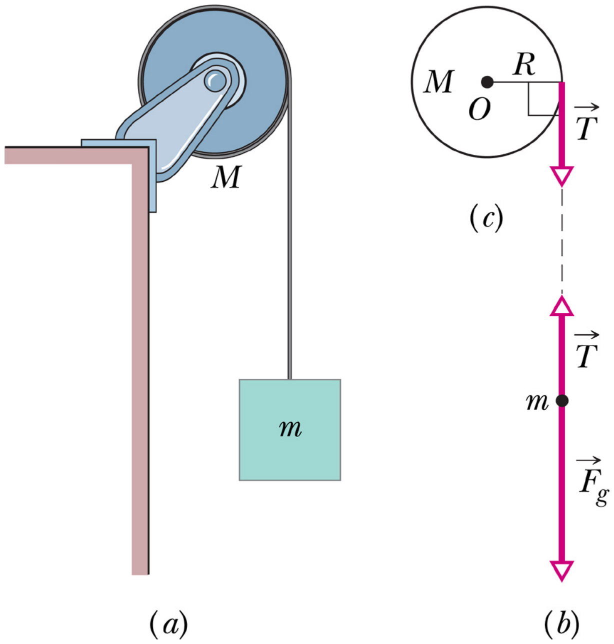
$$\omega = \frac{v}{r} = \frac{\left(2.90\times 10^4 \text{ km/h}\right)\left(1.000 \text{ h}/3600 \text{ s}\right)}{3.22\times 10^3 \text{ km}} = 2.50\times 10^{-3} \text{ rad/s.}$$

$$a_r = \omega^2 r = \left(2.50\times 10^{-3} \text{ rad/s}\right)^2 \left(3.22\times 10^6 \text{ m}\right) = 20.2 \text{ m/s}^2.$$

$$\alpha = \frac{d\omega}{dt} = 0 \text{ and } a_t = r\alpha = 0.$$

## Problem 3

The figure shows a uniform disk, with mass $M = 0.5kg$ and radius $R = 20cm$, mounted on a fixed horizontal axle. A block with mass $m = 2.5kg$ hangs from a massless cord that is wrapped around the rim of the disk. Find (a)the acceleration of the falling block, (b)the angular acceleration of the disk, and (c)the tension in the cord. The cord does not slip, and there is no friction at the axle.




(03小題)

(a)the magnitude of the acceleration of the block, $|a|$=________$m/s^2$

11: **ANS:=8.909**

(b)the magnitude of the angular acceleration of the disk, $|\alpha|$=__________$rad/s^2$

12: **ANS:=44.55**

(c)the tension in the cord=_______ N

13: **ANS:=2.227**

# Solution:

(a)block:$mg - T = ma$

disk:$\tau = rF_t = -RT$

$\tau_{net} = I\alpha$

$I = \frac{1}{2}MR^2$

$-RT = \frac{1}{2}MR^2\alpha$

$$a_t = R\alpha \rightarrow\rightarrow \alpha = \frac{a}{R}$$

$T = -\frac{1}{2}Ma$

$ma + mg = -\frac{1}{2}Ma$

$$a = -g\tfrac{2m}{M+2m} = -(9.8m/s^2)\frac{2(2.5kg)}{0.5kg + 2(2.5kg)} = 8.909m/s^2$$

(b)

$$\alpha = \frac{a}{R} = \frac{8.909m/s^2}{0.2m} = 44.545rad/s^2$$

(c)

$T = \frac{1}{2}Ma = \frac{1}{2}(0.5kg)(8.909m/s^2) = 2.227N$

## Problem 4

When a slice of buttered toast is accidentally pushed over the edge of a counter, it rotates as it falls. If the distance to the floor is $76cm$ and for rotation less than $1rev$, what are the (a) smallest and (b) largest angular speeds that cause the toast to hit and then topple to be butter-side down?

(02小題)

---

(a) smallest angular speed=_______rad/s

14: **ANS:=4**

(b) largest angular speed=_______rad/s

15: **ANS:=12**

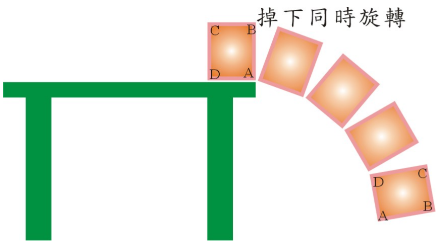
# Solution:

$h = \frac{1}{2}gt^2$

$$\Delta t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(0.76m)}{9.8}} = 0.394s$$

(a) 土司旋轉然後掉到地面時的最小角度為

$$\theta_{min} = 0.25rev = \frac{\pi}{2}rad$$




（掉到地面土司旋轉不超過一圈，且A端掉下，旋轉後B端撞到地面）

$$\omega_{min} = \frac{\delta_{min}}{\Delta t} = \frac{\frac{\pi}{2}rad}{0.394s} = 4rad/s$$

$$(b)\Delta\theta_{max} = 0.75rev = \frac{3\pi}{2}rad$$

（掉到地面土司旋轉不超過一圈，且A端掉下，旋轉後D端撞到地面）

$$\omega_{max}=\frac{\delta_{max}}{\Delta t}=\frac{\frac{3\pi}{2}rad}{0.394s}=12rad/s$$

## Problem 5

A diver makes 2.5 revolutions on the way from a $10m$ high platform to the water. Assuming zero initial vertical velocity, find the average angular velocity during the dive.

(01小題)

the average angular velocity=_______rad/s

16: **ANS:=11**

# Solution:

$$\Delta y = v_{0y}t - \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2\Delta y}{g}} = \sqrt{\frac{2(10m)}{9.8m/s^2}} = 1.4s$$

$$\omega_{avg} = \frac{(2.5rev)(2\pi rad/rev)}{1.4s} = 11rad/s$$

**Problem 6**
(a)Calculate the rotational inertia of a wheel that has a kinetic energy of $24400J$ when rotating at $602rev/min$. (01小題)

I=_______$kg.m^2$

17: **ANS:=12.3**

# Solution:

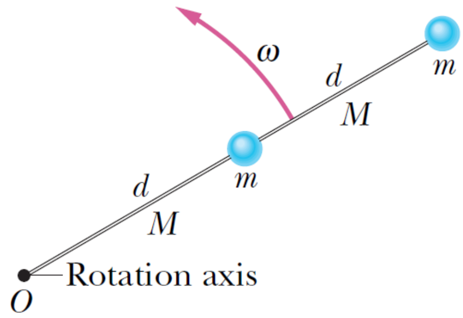
$K = \frac{1}{2}I\omega^2$

$$I = \frac{2K}{\omega^2} = \frac{2(24,400J)}{(\frac{(602rev/min)(2\pi rad/rev)}{60s/min})^2} = 12.29kg.m^2$$

nprob= 7 7

**Problem 7**

In the figure, two particles, each with mass m, are fastened to each other, and to a rotation axis at O, by two thin rods, each with length d and mass M_r. The combination rotates around the rotation axis with angular speed $\omega$=omega. Measured about O, what are the combination's (a) rotational inertia and (b) kinetic energy? (02 小題)




(a) rotational inertia=____ [m, M_r,d]

18: **ANS:=8/3*M_r*d**2+5*m*d**2**

(b) kinetic energy=_____ [m, M_r,d, omega]

19: **ANS:=(4/3*M_r+5/2*m)*d**2*omega**2**

# Solution:

$$I = I_1 + I_2 + I_3 + I_4 = \left(\frac{1}{12}Md^2 + M\left(\frac{1}{2}d\right)^2\right) + md^2 + \left(\frac{1}{12}Md^2 + M\left(\frac{3}{2}d\right)^2\right) + m(2d)^2$$

$$= \frac{8}{3}Md^2 + 5md^2 = \frac{8}{3}(1.2\text{ kg})(0.056\text{ m})^2 + 5(0.85\text{ kg})(0.056\text{ m})^2$$

$$= 0.023\text{ kg}\cdot\text{m}^2.$$

$$K = \frac{1}{2}I\omega^2 = \left(\frac{4}{3}M + \frac{5}{2}m\right)d^2\omega^2 = \left[\frac{4}{3}(1.2\text{ kg}) + \frac{5}{2}(0.85\text{ kg})\right](0.056\text{ m})^2(0.30\text{ rad/s})^2$$

$$= 1.1\times10^{-3}\text{ J}.$$

nprob= 7 7

**Problem 7**
Calculate the rotational inertia of a disk of mass M and radius R, about an axis perpendicular to the disk and passing through its center. (01小題)

I=______ [M,R]

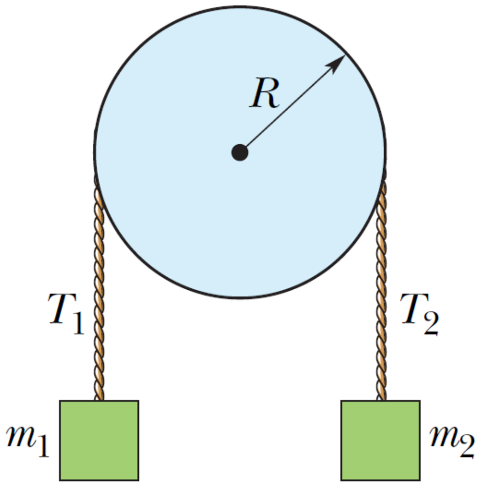
20: **ANS:=<u>1/2*m*R**2</u>**

# Solution:

$I = \frac{1}{2}MR^2 = \frac{1}{2}(2kg)(0.3m)^2 = 0.09kg \cdot m^2$
nprob= 8 8

**Problem 8**

In the figure, block 1 has mass m_1, block 2 has mass m_2, and the pulley, which is mounted on a horizontal axle with negligible friction, has radius R. When released from rest, block 2 falls a distance y in time t without the cord slipping on the pulley. (a) What is the magnitude of the acceleration of the blocks? What are (b) tension T_2 and (c) tension T_1? (d) What is the magnitude of the pulley's angular acceleration? (e) What is its rotational inertia? (05小題)




(a)the acceleration $a$=______ [y,t]

21: **ANS:=2*y/t**2**

(b)$T_2$=______ [m_2,y,t,g]

22: **ANS:=m_2*(g-2*y/t**2)**

(c)$T_1$=______ [m_1,y,t,g]

23: **ANS:=m_1*(g+2*y/t**2)**

(d)angular acceleration $\alpha$=_______ [y,t,R]

24: **ANS:=(2*y)/(R*t**2)**

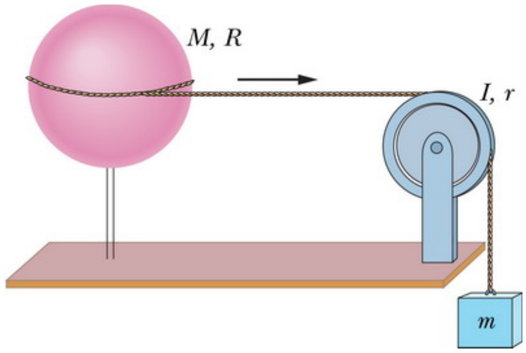
(e)moment of inertia of the pulley, I=______ [T_1,T_2,R,alpha] alpha=$\alpha$=angular acceleration

25: **ANS:=(T_2-T_1)*R/alpha**

nprob= 8 8

## Problem 8

A uniform spherical shell of mass $M = 4.5kg$ and radius $R = 8.5cm$ can rotate about a vertical axis on frictionless bearings (see figure). A massless cord passes around the equator of the shell, over a pulley of rotational inertia $I = 3.0 \times 10^{-3}\mathrm{kg.m^2}$ and radius $r = 5.0cm$, and is attached to a small object of mass $m = 0.60kg$.There is no friction on the pulley's axle; the cord does not slip on the pulley.What is the speed of the object when it has fallen $82cm$ after being released from rest? Use energy considerations.




(01小題)

v=_____ m/s

26: **ANS:=<u>1.42</u>**

# Solution:

球殼轉動慣量為$I_{shell} = \frac{2}{3}MR^2$

$$K = \frac{1}{2}(\frac{2}{3}MR^2)\omega_{sphere}^2 + \frac{1}{2}I\omega_{pulley}^2 + \frac{1}{2}mv^2 = mgh$$

$$\omega_{pully} = \frac{v}{r}$$

$$\omega_{sphere} = \frac{v}{R}$$

$$v = \sqrt{\frac{mgh}{\frac{1}{2}m + \frac{1}{2}\frac{I}{r^2} + \frac{M}{3}}}$$

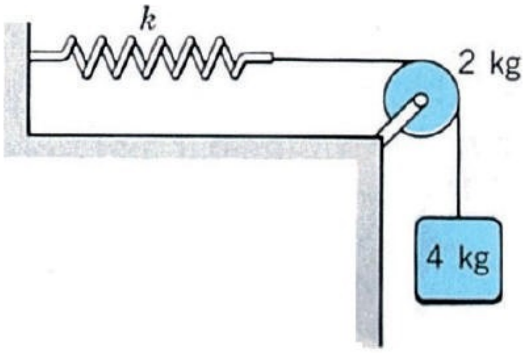
$$= \sqrt{\frac{2gh}{1 + \frac{I}{mr^2} + \frac{2M}{3m}}}$$

$$= \sqrt{\frac{2(9.8m/s^2)(0.82m)}{1 + \frac{3 \times 10^{-3} kg \cdot m^2}{(0.6kg)(0.05m)^2} + \frac{2(4.5kg)}{3(0.6kg)}}}$$

$$= 1.42m/s$$

## Problem 9

The figure shows a block of mass $4kg$ suspended by a rope that passes over a pulley of mass $2kg$ and radius 5cm. The rope is connect to a spring whose stiffness constant is $80N/m$. (a) If the block is released from rest, what is the maximum extension of the spring? (b) what is the speed of the block after it has fallen 20cm? Treat the pulley as a disk.




(02小題)

(a)x=_______m

27: **ANS:=0.98**

(b)v=_______m/s

28: **ANS:=1.58**

# Solution:

$$\Delta E = \frac{1}{2}mv^2 + \frac{1}{2}I(\frac{v}{R})^2 + \frac{1}{2}kx^2 - mgx = 0$$

(a)
$v = 0$

$$\Delta E = \frac{1}{2}kx^2 - mgx = 0$$

$$x = \frac{2mg}{k} = \frac{2(4kg)(9.8m/s^2)}{80N/m} = 0.98m$$

$$I = \frac{1}{2}mR^2 = \frac{1}{2}(2kg)(0.05m)^2 \rightarrow \rightarrow \frac{I}{R^2} = \frac{1}{2}m = \frac{1}{2}(2kg) = 1kg$$

(b)
$$\Delta E = \frac{1}{2}(4kg)v^2 + \frac{1}{2}(\frac{I}{R^2})v^2 + \frac{1}{2}(80N/m)(0.2m)^2 - (4kg)(9.8m/s^2)(0.2m) = 0$$

$v = 1.58m/s$