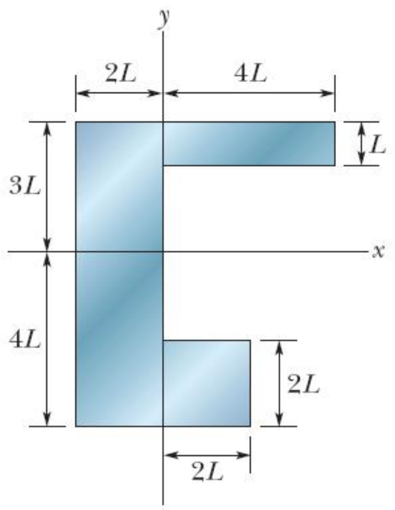
GP1-L09

Problem 1

What are (a) the x coordinate and (b) the y coordinate of the center of mass for the uniform plate shown in the figure if L=5 cm ?



(02小題)

(a)the x coordinate of the center of mass=___cm

01: **ANS:=-0.45**

(b)the y coordinate of the center of mass=___cm

02: **ANS:=-2**

Solution:

因為是uniform plate（均勻平版），所以質量與面積成正比。

第一像限：質量：$4L^2$。質量中心：(2L,2.5L)

第二像限：質量：$6L^2$。質量中心：(-L,1.5L)

第三像限：質量：$8L^2$。質量中心：(-L,-2L)

第四像限：質量：$4L^2$。質量中心：(L,-3L)

$$(a)x_{CM}=\frac{(4L^2)(2L)+(6L^2)(-L)+(8L^2)(-L)+(4L^2)(L)}{4L^2+6L^2+8L^2+4L^2}=\frac{-2}{22}L=-0.45cm$$

(b)

$$y_{CM} = \frac{(4L^2)(2.5L) + (6L^2)(1.5L) + (8L^2)(-2L) + (4L^2)(-3L)}{4L^2 + 6L^2 + 8L^2 + 4L^2} = \frac{-9}{22}L = -2cm$$

Problem 2

A fast athlete, of mass 70 kg, can run at 10 m/s. At what speed would the following have the same momentum: (a) a 20 g bullet; (b) a 1500 kg car?

(02小題)

(a)v=______m/s

03: **ANS:=3.5E4**

(b)v=______m/s

04: **ANS:=0.47**

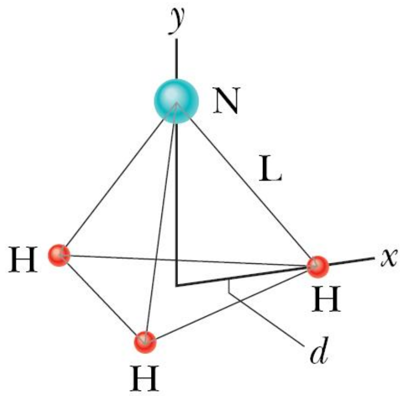
Solution:

$$(\text{a})v=\frac{P}{m}=\frac{(70kg)(10m/s)}{0.02kg}=3.5\times 10^4 m/s$$

$$(\text{b})v=\frac{P}{m}=\frac{(70kg)(10m/s)}{1500kg}=0.47m/s$$

Problem 3

In the ammonia (NH_3) molecule of the figure, three hydrogen (H) atoms form an equilateral triangle, with the center of the triangle at distance $d = 9.4 \times 10^{-11}$ m from each hydrogen atom. The nitrogen (N) atom is at the apex of a pyramid, with the three hydrogen atoms forming the base. The nitrogen-to-hydrogen atomic mass ratio is 13.9, and the nitrogen-to-hydrogen distance is $L = 10.14 \times 10^{-11}$ m. What are the (a) x and (b) y coordinates of the molecule's center of mass?



(02小題)

(a)x_{COM}=______ m

05: **ANS:=0**

(b)y_{COM}=______ m

06: **ANS:=3.13E-11**

Solution:

$$\frac{m_N}{m_H} = 13.9$$

(a)因為是對稱的，所以$x_{CM} = 0$

(b)$3m_h y_{CM} = m_N(y_N - y_{CM})$

其中y_N指的N原子到三個H原子平面的距離。

由圖可知，$y_N = \sqrt{(10.14 \times 10^{-11})^2 - (9.4 \times 10^{-11})^2} = 3.803 \times 10^{-11} m$

$$y_{CM} = \frac{m_N y_N}{m_N + 3m_H} = \frac{(13.9m_H)(3.803 \times 10^{-11})}{13.9m_H + 3(m_H)} = 3.13 \times 10^{-11} m$$

Problem 4

Ricardo, of mass 80 kg, and Carmelita, who is lighter, are enjoying Lake Merced at dusk in a 30 kg canoe. When the canoe is at rest in the placid water, they exchange seats, which are 3 m apart and symmetrically located with respect to the canoe's center. If the canoe moves 40 cm horizontally relative to a pier post, what is Carmelita's mass?

(01小題)

m_C=______kg

07: **ANS:=58**

Solution:

$$m_R(\frac{L}{2}-x)=mx+m_C(\frac{L}{2}+x)$$

$$x=\frac{40cm}{2}=20cm=0.2m$$

$$m_C=\frac{m_R(\frac{L}{2}-x)-mx}{\frac{L}{2}+x}=\frac{(80kg)(\frac{3}{2}-0.2)-(30kg)(0.2)}{\frac{3}{2}+0.2}=58kg$$

Problem 5

A 0.70 kg ball moving horizontally at 6.0 m/s strikes a vertical wall and rebounds with speed 3.5 m/s. What is the magnitude of the change in its linear momentum?

(01小題)

linear momentum p=______kg·m/s

08: **ANS:=6.7**

Solution:

$$\Delta p = m|v_i - v_f| = (0.7kg)|6m/s - (-3.5m/s)| = 6.7kg \times m/s$$

Problem 6

A 91 kg man lying on a surface of negligible friction shoves a 68 g stone away from himself, giving it a speed of 4 m/s. What speed does the man acquire as a result?

(01小題)

speed v=_______m/s

09: **ANS:=3E-3**

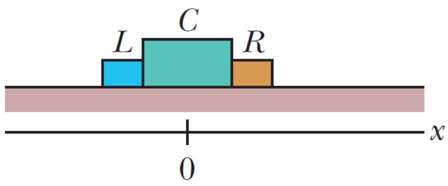
Solution:

$$m_m v_m = m_s v_s$$

$$v_m = \frac{m_s v_s}{m_m} = \frac{(0.068kg)(4m/s)}{91kg} = 3 \times 10^{-3} m/s$$

Problem 7

The figure shows a two-ended “rocket” that is initially stationary on a frictionless floor, with its center at the origin of an x axis.The rocket consists of a central block C (of mass M = 6.00 kg) and blocks L and R (each of mass m = 2.00 kg) on the left and right sides. Small explosions can shoot either of the side blocks away from block C and along the x axis. Here is the sequence: (1) At time t = 0, block L is shot to the left with a speed of 3.00 m/s relative to the velocity that the explosion gives the rest of the rocket. (2) Next, at time t = 0.80 s, block R is shot to the right with a speed of 3.00 m/s relative to the velocity that block C then has. At t = 2.80 s, what are (a) the velocity of block C and (b) the position of its center? Notice the sign of your answers according to the coorinates of this system. (02小題)



(a)v_C=______ m/s

10: **ANS:=-0.15**

(b)x_C=______ m

11: **ANS:=0.18**

Solution:

(a) With SI units understood, the velocity of block L (in the frame of reference indicated in the figure that goes with the problem) is $(v_1 - 3)\hat{\mathrm{i}}$. Thus, momentum conservation (for the explosion at $t = 0$) gives

$$m_L(v_1 - 3) + (m_C + m_R)v_1 = 0$$

which leads to

$$v_1 = \frac{3\,m_L}{m_L + m_C + m_R} = \frac{3(2\text{ kg})}{10\text{ kg}} = 0.60\text{ m/s}.$$

Next, at $t = 0.80$ s, momentum conservation (for the second explosion) gives

$$m_C\,v_2 + m_R(v_2 + 3) = (m_C + m_R)v_1 = (8\text{ kg})(0.60\text{ m/s}) = 4.8\text{ kg}\cdot\text{m/s}.$$

This yields $v_2 = -0.15$. Thus, the velocity of block C after the second explosion is

$$v_2 = -(0.15\text{ m/s})\hat{\mathrm{i}}.$$

(b) Between $t = 0$ and $t = 0.80$ s, the block moves $v_1\Delta t = (0.60\text{ m/s})(0.80\text{ s}) = 0.48$ m. Between $t = 0.80$ s and $t = 2.80$ s, it moves an additional

$$v_2\Delta t = (-0.15\text{ m/s})(2.00\text{ s}) = -0.30\text{ m}.$$

Its net displacement since $t = 0$ is therefore 0.48 m – 0.30 m = 0.18 m.

Problem 8

A vessel at rest at the origin of an xy coordinate system explodes into three pieces. Just after the explosion, one piece, of mass m, moves with velocity (-30 m/s)$\hat{i}$ and a second piece, also of mass m, moves with velocity (-30 m/s) $\hat{j}$. The third piece has mass 3m. Just after the explosion, what are the the velocity($\vec{v}_3 = v_x\hat{i} + v_y\hat{j}$ of the third piece? (02小題)

(a)v_x=_____ m/s

12: **ANS:=<u>10</u>**

(b)v_y=_____ m/s

13: **ANS:=<u>10</u>**

Solution:

Our notation (and, implicitly, our choice of coordinate system) is as follows: the mass of one piece is $m_1 = m$; its velocity is $\vec{v}_1 = (-30\text{ m/s})\hat{i}$; the mass of the second piece is $m_2 = m$; its velocity is $\vec{v}_2 = (-30\text{ m/s})\hat{j}$; and, the mass of the third piece is $m_3 = 3m$.

(a) Conservation of linear momentum requires

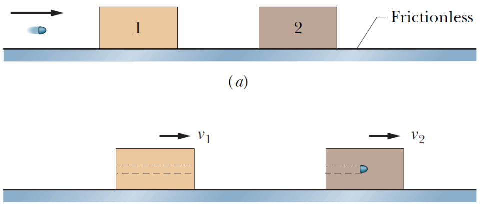
$$m\vec{v}_0 = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 \quad \Rightarrow \quad 0 = m\left(-30\hat{i}\right) + m\left(-30\hat{j}\right) + 3m\vec{v}_3$$

which leads to $\vec{v}_3 = (10\hat{i} + 10\hat{j})$ m/s. Its magnitude is $v_3 = 10\sqrt{2} \approx 14$ m/s.

(b) The direction is 45° *counterclockwise* from $+x$ (in this system where we have m_1 flying off in the $-x$ direction and m_2 flying off in the $-y$ direction).

Problem 9

In the figure, a 3.50 g bullet is fired horizontally at two blocks at rest on a frictionless table. The bullet passes through block 1 (mass 1.20 kg) and embeds itself in block 2 (mass 1.80 kg). The blocks end up with speeds $v_1 = 0.630$ m/s and $v_2 = 1.40$ m/s. Neglecting the material removed from block 1 by the bullet, find the speed of the bullet as it (a) leaves and (b) enters block 1. (02小題)



(a)the speed of the bullet as it leaves block 1=____ m/s

14: **ANS:=721**

(b)the speed of the bullet as it enter block 1=____ m/s

15: **ANS:=937**

Solution:

53. In solving this problem, our $+x$ direction is to the right (so all velocities are positive-valued).

(a) We apply momentum conservation to relate the situation just before the bullet strikes the second block to the situation where the bullet is embedded within the block.

$$(0.0035 \text{ kg})v = (1.8035 \text{ kg})(1.4 \text{ m/s}) \Rightarrow v = 721 \text{ m/s}.$$

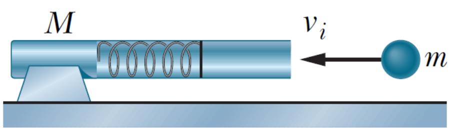
(b) We apply momentum conservation to relate the situation just before the bullet strikes the first block to the instant it has passed through it (having speed v found in part (a)).

$$(0.0035 \text{ kg})v_0 = (1.20 \text{ kg})(0.630 \text{ m/s}) + (0.00350 \text{ kg})(721 \text{ m/s})$$

which yields $v_0 = 937$ m/s.

Problem 10

In the figure, a ball of mass m = 60 g is shot with speed vi = 22 m/s into the barrel of a spring gun of mass M = 240 g initially at rest on a frictionless surface.The ball sticks in the barrel at the point of maximum compression of the spring. Assume that the increase in thermal energy due to friction between the ball and the barrel is negligible. (a) What is the speed of the spring gun after the ball stops in the barrel? (b)What fraction of the initial kinetic energy of the ball is stored in the spring? (02小題)



(a)the speed of the spring gun after the ball stops=________ m/s

16: **ANS:=<u>4.4</u>**

(b)the fraction of the initial kinetic energy of the ball is stored in the spring=______

17: **ANS:=<u>0.8</u>**

Solution:

(a) Let v be the final velocity of the ball-gun system. Since the total momentum of the system is conserved $mv_i = (m + M)v$. Therefore,

$$v = \frac{mv_i}{m+M} = \frac{(60\ \text{g})(22\ \text{m/s})}{60\ \text{g} + 240\ \text{g}} = 4.4\ \text{m/s}.$$

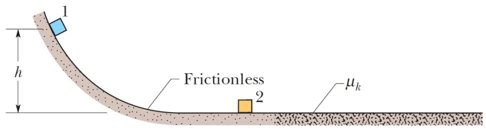
(b) The initial kinetic energy is $K_i = \frac{1}{2}mv_i^2$ and the final kinetic energy is $K_f = \frac{1}{2}(m+M)v^2 = \frac{1}{2}m^2v_i^2/(m+M)$. The problem indicates $\Delta E_{\text{th}} = 0$, so the difference $K_i - K_f$ must equal the energy U_s stored in the spring:

$$U_s = \frac{1}{2}mv_i^2 - \frac{1}{2}\frac{m^2v_i^2}{(m+M)} = \frac{1}{2}mv_i^2\left(1 - \frac{m}{m+M}\right) = \frac{1}{2}mv_i^2\frac{M}{m+M}.$$

Consequently, the fraction of the initial kinetic energy that becomes stored in the spring is

$$\frac{U_s}{K_i} = \frac{M}{m+M} = \frac{240}{60+240} = 0.80.$$

Problem 11



In the figure, block 1 of mass m_1 slides from rest along a frictionless ramp from height h = 2.50 m and then collides with stationary block 2, which has mass m_2 = 2m_1. After the collision, block 2 slides into a region where the coefficient of kinetic friction μ_k is 0.500 and comes to a stop in distance d within that region. What is the value of distance d if the collision is (a) elastic and (b) completely inelastic? (02小題)

(a)for elastic collision, d=______ m

18: **ANS:=2.22**

(b)for inelastic collision, d=______ m

19: **ANS:=0.56**

Solution:

(a) If the collision is perfectly elastic, then Eq. 9-68 applies

$$v_2 = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2m_1}{m_1 + (2.00)m_1}\sqrt{2gh} = \frac{2}{3}\sqrt{2gh}$$

where we have used the fact (found most easily from energy conservation) that the speed of block 1 at the bottom of the frictionless ramp is $\sqrt{2gh}$ (where h = 2.50 m). Next, for block 2's "rough slide" we use Eq. 8-37:

$$\frac{1}{2} m_2 v_2^2 = \Delta E_{th} = f_k d = \mu_k m_2 g\, d\,.$$

where μ_k = 0.500. Solving for the sliding distance d, we find that m_2 cancels out and we obtain d = 2.22 m.

(b) In a completely inelastic collision, we apply Eq. 9-53: $v_2 = \frac{m_1}{m_1 + m_2} v_{1i}$ (where, as above, $v_{1i} = \sqrt{2gh}$). Thus, in this case we have $v_2 = \sqrt{2gh}/3$. Now, Eq. 8-37 (using the total mass since the blocks are now joined together) leads to a sliding distance of $d = 0.556$ m (one-fourth of the part (a) answer).

Problem 12

A projectile proton with a speed of 500 m/s collides elastically with a target proton initially at rest. The two protons then move along perpendicular paths, with the projectile path at 60° from the original direction. After the collision, what are the speeds of (a) the target proton and (b) the projectile proton? Proton mass=1.67×10^{-27} kg. (02小題)

(a)the speeds of the target proton=______ m/s

20: **ANS:=433**

(b)the speeds of the projectile proton=_______ m/s

21: **ANS:=250**

Solution:

71. We orient our +x axis along the initial direction of motion, and specify angles in the "standard" way — so $\theta = +60°$ for the proton (1) which is assumed to scatter into the first quadrant and $\phi = -30°$ for the target proton (2) which scatters into the fourth quadrant (recall that the problem has told us that this is perpendicular to θ). We apply the conservation of linear momentum to the x and y axes respectively.

$$\begin{aligned} m_1 v_1 &= m_1 v'_1 \cos\theta + m_2 v'_2 \cos\phi \\ 0 &= m_1 v'_1 \sin\theta + m_2 v'_2 \sin\phi \end{aligned}$$

We are given v_1 = 500 m/s, which provides us with two unknowns and two equations, which is sufficient for solving. Since $m_1 = m_2$ we can cancel the mass out of the equations entirely.

(a) Combining the above equations and solving for v'_2 we obtain

$$v'_2 = \frac{v_1 \sin\theta}{\sin(\theta - \phi)} = \frac{(500 \text{ m/s})\sin(60°)}{\sin(90°)} = 433 \text{ m/s}.$$

We used the identity $\sin\theta\cos\phi - \cos\theta\sin\phi = \sin(\theta - \phi)$ in simplifying our final expression.

(b) In a similar manner, we find

$$v'_1 = \frac{v_1 \sin\theta}{\sin(\phi - \theta)} = \frac{(500 \text{ m/s})\sin(-30°)}{\sin(-90°)} = 250 \text{ m/s}.$$