GP1-L08

Problem 1

What is the spring constant of a spring that stores 27J of elastic potential energy when compressed by 8.5cm? (01小題)

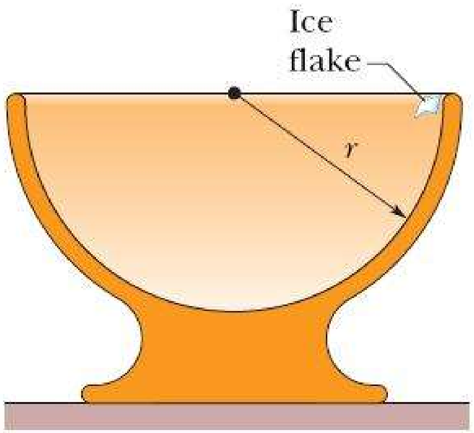
彈性係數為=_______N/m

$$位能U = \frac{1}{2}kx^2$$

$$k = \frac{2U}{x^2} = \frac{2(27J)}{(0.085m)^2} = 7.47 \times 10^3 N/m$$

Problem 2

In the figure, a 2 g ice flake is released from the edge of a hemispherical bowl whose radius r is 22 cm. The flake–bowl contact is frictionless. (a) How much work is done on the flake by the gravitational force during the flake's descent to the bottom of the bowl? (b) What is the change in the potential energy of the flake–Earth system during that descent? (c) If that potential energy is taken to be zero at the bottom of the bowl, what is its value when the flake is released? (d) If, instead,the potential energy is taken to be zero at the release point, what is its value when the flake reaches the bottom of the bowl? (e) If the mass of the flake were doubled, would the magnitudes of the answers to (a) through (d) increase, decrease, or remain the same?



(a)$W = mgh = (2 \times 10^{-3} kg)(9.8 m/s^2)(22 \times 10^{-2} m) = 4.31 \times 10^{-3} J$

(b)$\Delta U = -W = -4.31 \times 10^{-3} J$

(c)$U = +4.31 \times 10^{-3} J$

(d)在頂端時$U = 0$，則在碗底部時$U = -4.31 \times 10^{-3} J$

(e)$W = mgh$，質量m變2倍，所有答案也會變2倍。

(a)W=______J

02: **ANS:=4.31E-3**

(b)ΔU=______J

03: **ANS:=-4.31E-3**

(c)U=______J

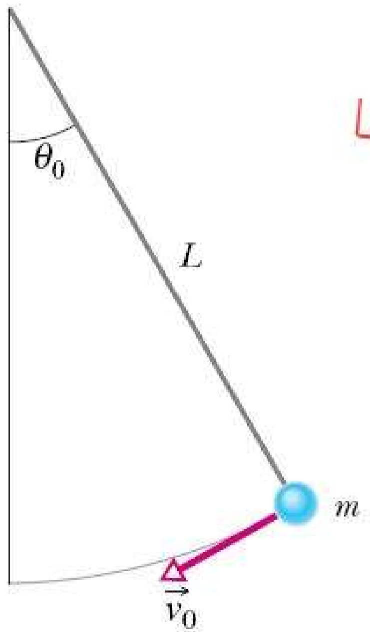
04: **ANS:=4.31E-3**

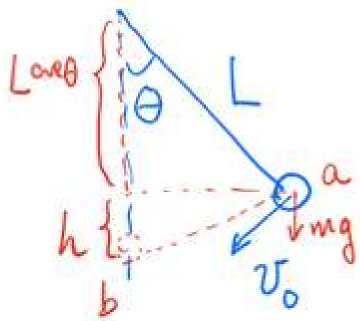
(d)U=______J

05: **ANS:=-4.31E-3**

Problem 3

The figure shows a thin rod, of length $L = 2$ m and negligible mass, that can pivot about one end to rotate in a vertical circle. A ball of mass $m = 5$ kg is attached to the other end. The rod is pulled aside to angle $\theta = 30^0$ and released with initial velocity $\vec{v}_0 = 0$. As the ball descends to its lowest point, (a) how much work does the gravitational force do on it and (b) what is the change in the gravitational potential energy of the ball –Earth system? (c) If the gravitational potential energy is taken to be zero at the lowest point, what is its value just as the ball is released? (d) Do the magnitudes of the answers to (a) through (c) increase, decrease, or remain the same if angle θ is increased?





$h = L - L\cos\theta$

$W_g = mgh = mgL(1-\cos\theta)$

$= 5(9.8)(2)(1-\cos 30^\circ)$

$= 13.13$ (J)

$\Delta U = -W_g = -13.13$

$-13.13 = U_b - U_a = 0 - U_a$

$U_a = 13.13$

$(-13.13 = U_b - 0 \Rightarrow U_b = -13.13)$

$\theta\uparrow \Rightarrow h\uparrow$, $W_g, |\Delta U|, U_a \uparrow$

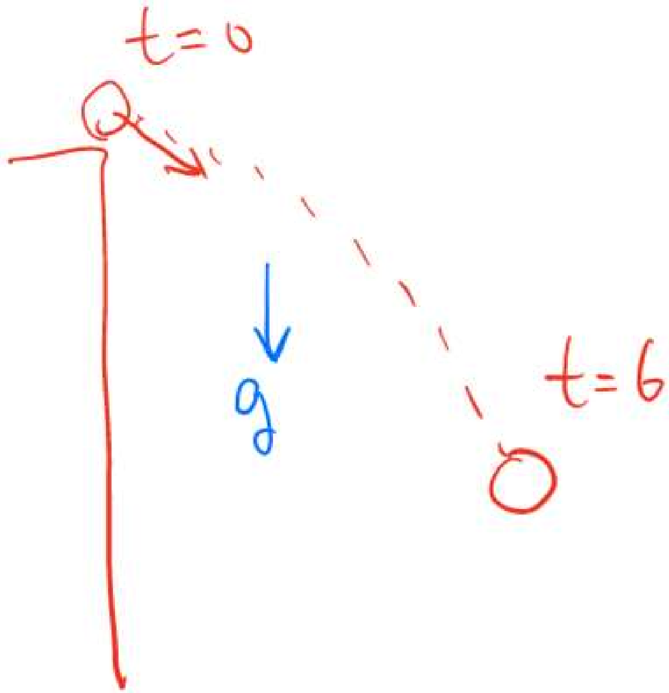
Problem 4

At $t = 0$ a 1.0 kg ball is thrown from a tall tower with $\vec{v} = 18\hat{i} + 24\hat{j}$ m/s. What is ΔU of the ball–Earth system between $t = 0$ and $t = 6$ s (still free fall)? (01小題)

(01小題)

ΔU=______J

11: **ANS:=-317.5**



力學能守恆：
位能的變化量
= - (動能變化量)

$$\vec{v} = \vec{v}_0 + (-9.8\hat{j})t$$

$$= (18, 24) + (-9.8)(6)$$

$$= (18, -34.8)$$

$$|\vec{v}| = 39.18, \ |\vec{v}_0| = 30$$

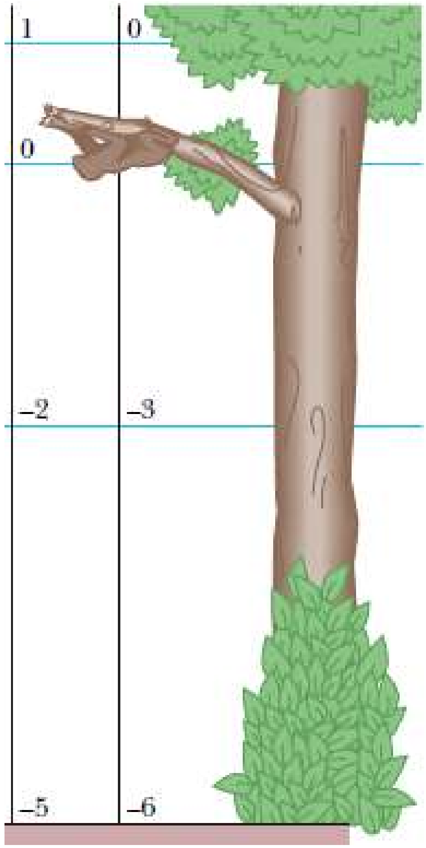
$$E = \text{const.}$$

$$\Delta U = -\Delta K = \frac{1}{2}(1)(30^2 - 39.18^2)$$

$$= -317.5$$

Problem 5

A 3.2 kg sloth hangs 3.0 m above the ground. (a) What is the gravitational potential energy of the sloth –Earth system if we take the reference point $y = 0$ to be at the ground? If the sloth drops to the ground and air drag on it is assumed to be negligible, what are the (b) kinetic energy and (c) speed of the sloth just before it reaches the ground? (03小題)



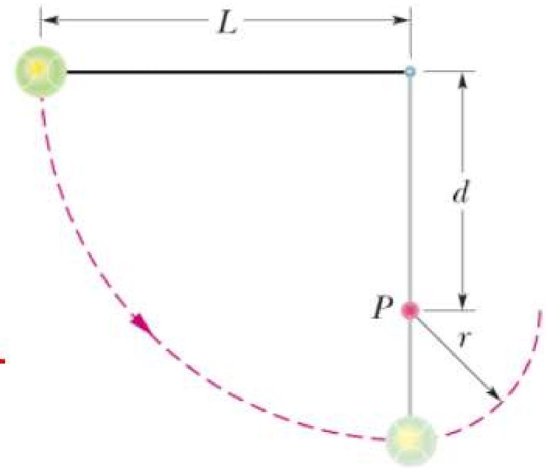
(a)$U = mgh = (3.2kg)(9.8m/s^2)(3m) = 94J$

(b)The mechanical energy is conserved, so $K = 94J$.

(c)$v = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2(94J)}{32kg}} = 7.7m/s$

Problem 6

The length of the string in the figure is L, has a ball attached to one end, and is fixed at its other end. The distance from the fixed end to a fixed peg at point P is d ($> L/2$). When the initially stationary ball is released with the string horizontal as shown, it will swing along the dashed arc. What is its speed when it reaches (a) its lowest point(v_L) and (b) its highest point after the string catches on the peg(v_P)? (02小題)



(a) v_L=____ [g,L]

15: **ANS:=sqrt(2*g*L)**

$$mgL = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gL} \ .$$

(b) v_P=____ [g,L,d]

16: **ANS:=sqrt(4*g*d-2*g*L)**

$$mgL = \frac{1}{2}mv_b^2 + mgy_b$$

$$v_b = \sqrt{2gL - 2g(2r)} \qquad r = L - d$$

$$v_b = \sqrt{4gd - 2gL}$$

如果擺錘要完成完整的圓週運動，也就是擺錘運動到釘子的正上方時，繩子仍然沒有鬆掉，請問釘子P的距離d必須滿足的條件？

$$mg = \frac{mv_t^2}{r}$$

$$v_t \geq \sqrt{gr}$$

$$\sqrt{4gd - 2gL} \geq 0$$

$$\Rightarrow d \geq \frac{L}{2}$$

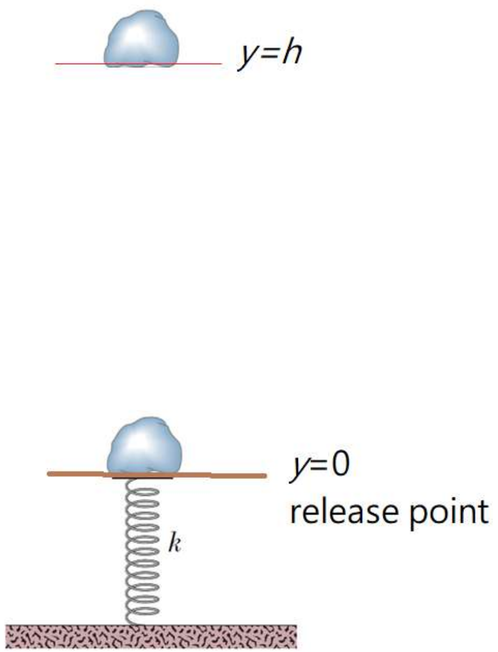
$$\sqrt{4gd - 2gL} \geq \sqrt{g(L-d)}$$

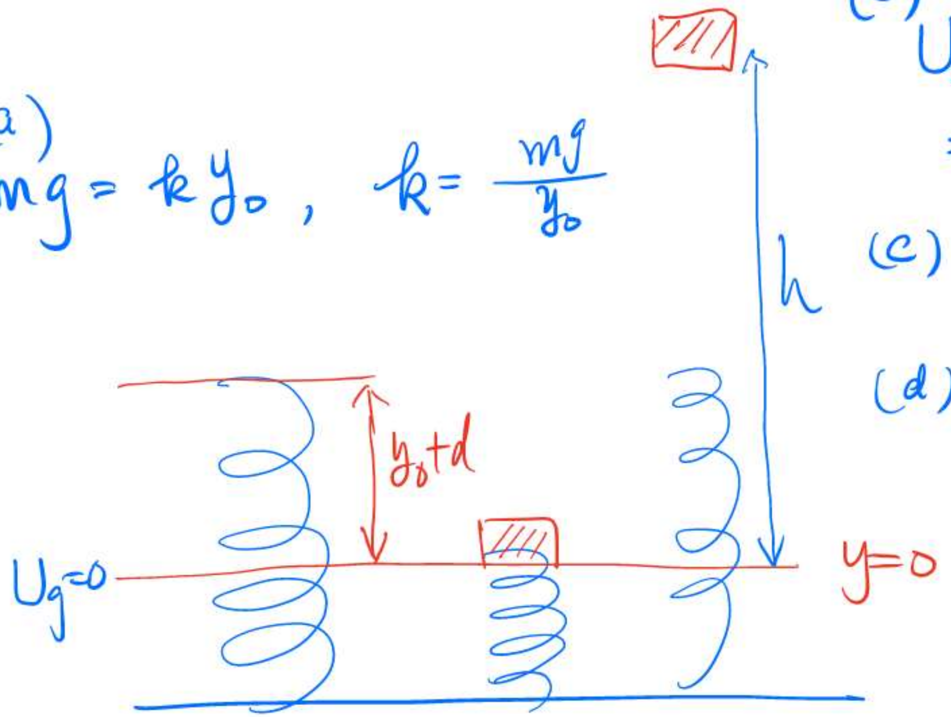
$$4gd - 2gL \geq gL - gd$$

$$5d \geq 3L, \ \boxed{d \geq \frac{3}{5}L} > \frac{L}{2}$$

Problem 7

The figure shows a stone of mass m at rest on a spring. The spring is compressed y_0 by the stone. (a) What is the spring constant k? (b) The stone is pushed down an additional distance d and released. What is the elastic potential energy(U_{sp}) of the compressed spring just before that release? (c) What is the change in the gravitational potential energy(ΔU_g) of the stone–Earth system when the stone moves from the release point to its maximum height? (d) What is that maximum height(h), measured from the release point? (04小題)





(a)
$$mg = k y_0, \quad k = \frac{mg}{y_0}$$

(b)
$$U_{sp} = \frac{1}{2}k(y_0+d)^2 = \frac{1}{2}\left(\frac{mg}{y_0}\right)(y_0+d)^2$$

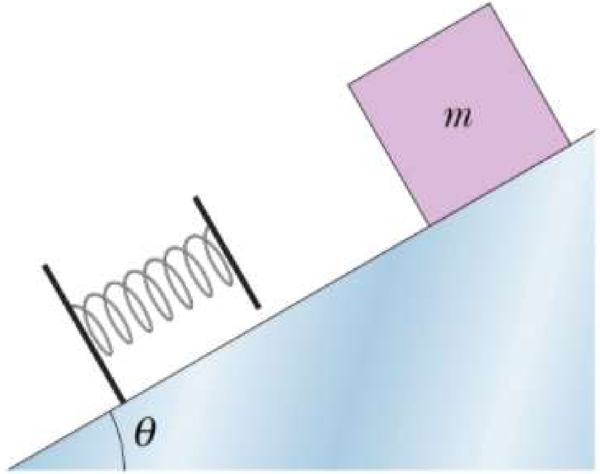
(c) $U_g = U_{sp}$

(d)
$$mgh = \frac{1}{2}\left(\frac{mg}{y_0}\right)(y_0+d)^2$$

$$h = \frac{(y_0+d)^2}{2y_0}$$

Problem 8

In the figure, a block of mass m is released from rest on a frictionless incline of angle θ=theta. Below the block is a spring with spring constant k. The block momentarily stops when it compresses the spring by x. (a) How far does the block move down the incline from its rest position to this stopping point? (b) What is the speed of the block just as it touches the spring? (01小題)

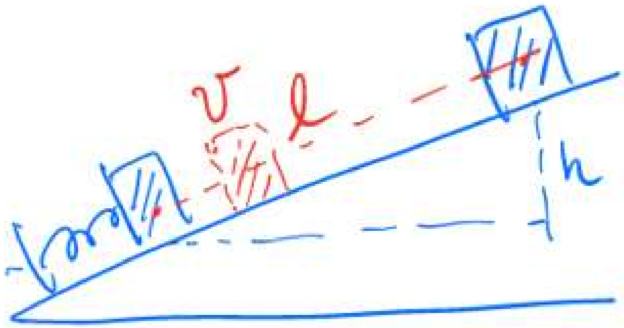


the distance the block moves down=____ [k,x,m,g,theta]

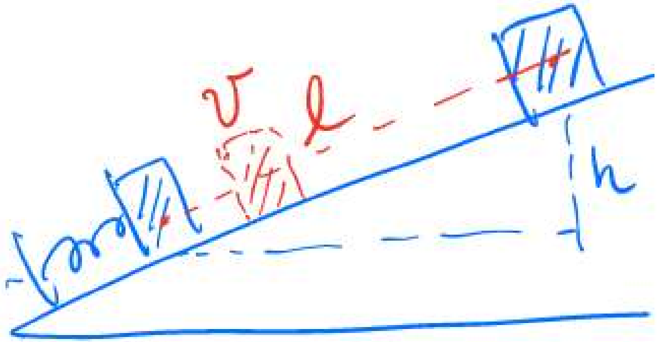
21: **ANS:=(k*x**2)/(2*m*g*sin(theta))**

(b)speed, v=______ [m,g,x,k,theta]

22: **ANS:=sqrt(0.5*k*x**2-m*g*x*sin(theta))**



比較出發點和最大壓縮量這兩個點，在這兩個位置的狀態都是靜止，速度為零，動能都是0，表示動能變化量為0。因此總位能的變化量也必須是0，所以重力位能的變化量應該等於負的彈性位能的變化量。



$$mgh = \frac{1}{2}kx^2$$

$$h = \frac{kx^2}{2mg}$$

$$l = \frac{h}{\sin\theta} = \frac{kx^2}{2mg\sin\theta}$$

$$\frac{1}{2}mv^2 = mg(l-x)\sin\theta$$

$$v = \sqrt{mgl\sin\theta - mgx\sin\theta}$$

$$= \sqrt{mgh - mgx\sin\theta}$$

$$= \sqrt{\frac{1}{2}kx^2 - mgx\sin\theta}$$

Problem 9

The potential energy of a diatomic molecule (a two atom system like H_2 or O_2) is given by

$$U = \frac{A}{r^{12}} - \frac{B}{r^6}$$

保守力可以透過對位能
函數微分得到

where r is the separation of the two atoms of the molecule and A and B are positive constants. This potential energy is associated with the force that binds the two atoms together. Find the equilibrium separation—that is, the distance between the atoms at which the force on each atom is zero. (01小題)

The force at the equilibrium position $r = r_{eq}$ is

the equilibrium separation, r_{eq}=_____ [A,B]

$$F = -\left.\frac{dU}{dr}\right|_{r = r_{eq}} = 0 \quad \Rightarrow \quad -\frac{12A}{r_{eq}^{13}} + \frac{6B}{r_{eq}^{7}} = 0$$

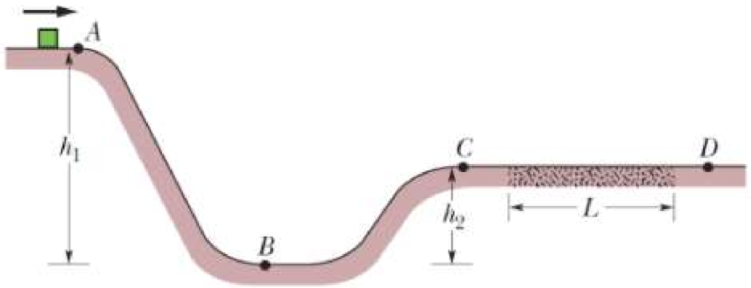
23: **ANS:=1.12*(A/B)**(1/6)**

which leads to the result

$$r_{eq} = \left(\frac{2A}{B}\right)^{\frac{1}{6}} = 1.12\left(\frac{A}{B}\right)^{\frac{1}{6}}.$$

Problem 10

In the figure, a small block of mass m is sent through point A with a speed of v. Its path is without friction until it reaches the section of length L, where the coefficient of kinetic friction is μ_k. The indicated heights are h_1 and h_2. What are the speeds of the block at (a) point B and (b) point C? (c) If the block can reach point D, what is its speed there; if not, how far through the section of friction does it travel? (03小題)



(a)v_B=______ [m,v,g,h_1,h_2,mu_k]

24: **ANS:=sqrt(v**2+2*g*h_1)**

(b)v_C=______ [m,v,g,h_1,h_2,mu_k]

25: **ANS:=sqrt(v**2+2*g*(h_1-h_2))**

(c)d=______ [m,v,g,h_1,h_2,mu_k]

26: **ANS:=(v**2+2*g*(h_1-h_2))/(2*mu_k*g)**

at B, $\frac{1}{2}mv_B^2 = \frac{1}{2}mv^2 + mgh_1$

$v_B = \sqrt{v^2+2gh_1}$

at C, $\frac{1}{2}mv_C^2 = \frac{1}{2}mv^2 + mg(h_1-h_2)$

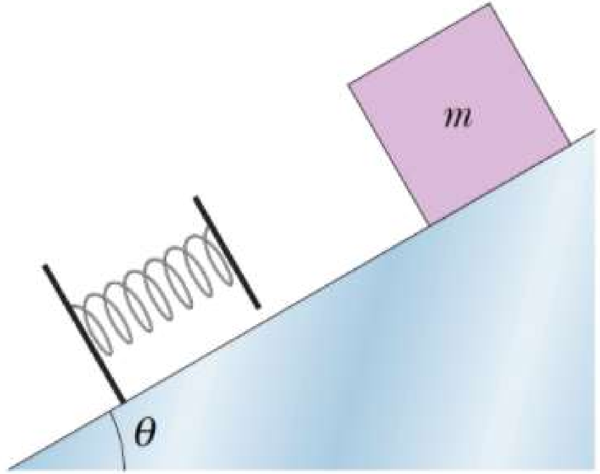
$v_C = \sqrt{v^2+2g(h_1-h_2)}$

stop : $\frac{1}{2}mv_C^2 = (\mu_k mg)d$

$d = \frac{v_C^2}{2\mu_k g} = \frac{v^2+2g(h_1-h_2)}{2\mu_k g}$

Problem 8

In the figure, a block of mass m is released from rest on a frictionless incline of angle θ=theta. Below the block is a spring with spring constant k. The block momentarily stops when it compresses the spring by x. (a) How far does the block move down the incline from its rest position to this stopping point? (b) What is the speed of the block just as it touches the spring? (01小題)



the distance the block moves down=____ [k,x,m,g,theta]

21: **ANS:=(k*x**2)/(2*m*g*sin(theta))**

$$K_A + U_A = K_C + U_C$$

$$0 + mgh = 0 + \frac{1}{2}kx^2$$

$$k = \frac{F}{x} = \frac{270\ \text{N}}{0.02\ \text{m}} = 1.35\times10^4\ \text{N/m}\,.$$

$$h = \frac{kx^2}{2mg} = \frac{(1.35\times10^4\ \text{N/m})(0.055\ \text{m})^2}{2(12\ \text{kg})(9.8\ \text{m/s}^2)} = 0.174\ \text{m}\,.$$

$$\frac{h}{\ell + |x|} = \sin\theta$$

$$\ell = 0.35 - 0.055 = 0.29\ \text{m}$$

$$\Delta y = -\ell\sin\theta = -0.15\ \text{m}$$

$$\Delta K + \Delta U = 0$$

$$\frac{1}{2}mv_B^2 + mg\Delta h = 0$$

$$v_B = \sqrt{-2g\Delta h}$$

$$= \sqrt{-(9.8)(-0.15)}$$

$$= 1.7\ \text{m/s}\,.$$

(b)speed, v=______ [m,g,x,k,theta]

$$\ell = \frac{0.174\ \text{m}}{\sin 30^\circ} = 0.35\ \text{m}\,.$$

22: **ANS:=sqrt(0.5*k*x**2-m*g*x*sin(theta))**