

GP1-L03

Problem 1

What are (a) the x component and (b) the y component of a vector **a** in the xy plane if its direction is 250^0 counterclockwise from the positive direction of the x axis and its magnitude as $7.3m$? 若向量 **a** 在 xy 平面上的方向是從 x 的正方向逆時針旋轉 250^0 ，則 (a) x 分量和 (b) y 分量分別是什麼軸及其大小為 $7.3m$ ？

(02小題)

(a)_____m

01: ANS:=-2.5

(b)_____m

02: ANS:=-6.9

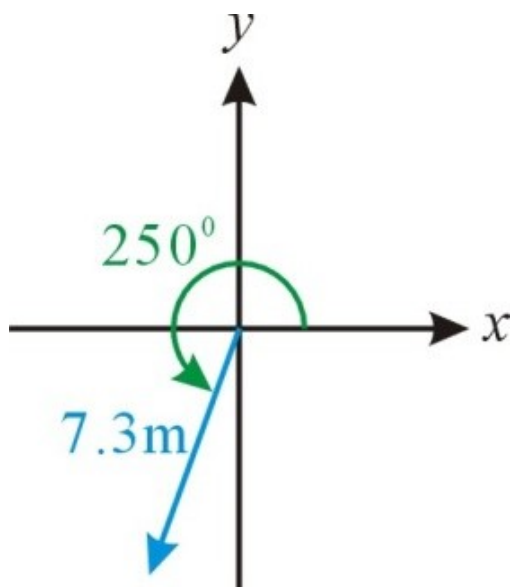
Solution:

(a)

$$a_x = a \cos \theta = (7.3m)(\cos 250^0) = -2.5m$$

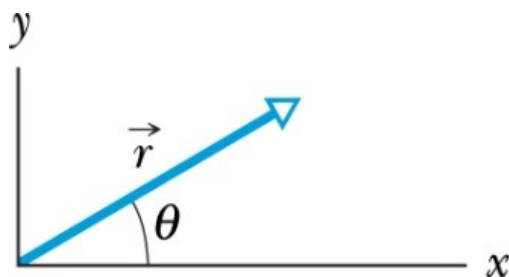
(b)

$$a_y = a \sin \theta = (7.3m)(\sin 250^0) = -6.9m$$



Problem 2

A displacement vector $\vec{r}$ in the xy plane is $12m$ long and directed at angle $\theta = 30^\circ$ in the figure. Determine (a) the x component and (b) the y component of the vector. xy 平面上的位移向量 $\vec{r}$ 長 $12m$ ，方向為圖中的角 $\theta = 30^\circ$ 。求向量的 (a) x 分量和 (b) y 分量。



(02小題)

(a) x component = _____ m

03: ANS: = 10.4

(b) y component = _____ m

04: ANS: = 6

Solution:

(a)

$$r_x = r \cos \theta = (12m)(\cos 30^\circ) = 10.4m$$

(b)

$$r_y = r \sin \theta = (12m)(\sin 30^\circ) = 6m$$

Problem 3

The x component of vector $\vec{A}$ is $-25m$ and the y component is $+40.0m$.
 (a) What is the magnitude of $\vec{A}$? (b) What is the angle between the direction of $\vec{A}$ and the positive direction of x? 向量 $\vec{A}$ 的 x 分量為 $-25m$ ，y 分量為 $+40.0m$ 。 (a) $\vec{A}$ 的大小是多少？ (b) $\vec{A}$ 的方向與 x 的正方向之間的夾角是多少？

(02小題)

(a) $A = \underline{\hspace{2cm}} m$

05: ANS: = 47.2

(b) the angle = $\underline{\hspace{2cm}}$

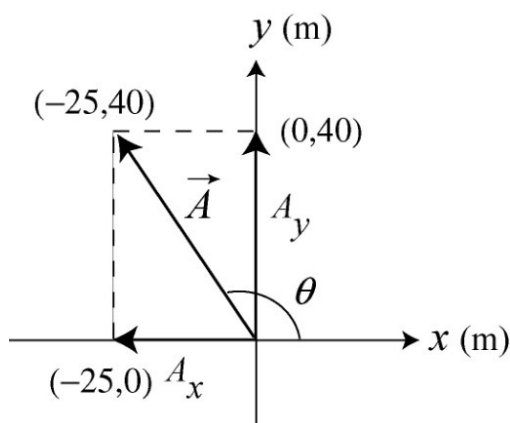
06: ANS: = 122

Solution:

$$(a) A = \sqrt{(-25m)^2 + (40m)^2} = 47.2m$$

$$(b) \tan^{-1} \frac{40m}{-25m} = -58^\circ \text{ or } 122^\circ$$

$(-25m, 40m)$ 在第二象限， $\tan \theta = \tan(\theta + 180^\circ)$ ，所以選 122 度



Problem 4

If $\vec{B}$ is added to $\vec{C} = 3\hat{i} + 4\hat{j}$, the result is a vector in the positive direction of the y axis, with a magnitude equal to that of $\vec{C}$. What is the magnitude of $\vec{B}$? 如果將 $\vec{B}$ 與 $\vec{C} = 3\hat{i} + 4\hat{j}$ 相加，則結果是 y 軸正方向的向量，其幅度等於 $\vec{C}$ 的幅度。 $\vec{B}$ 的大小是多少？

(01小題)

the magnitude of $\vec{B}$ = ____

07: ANS:=3.2

Solution:

$$\vec{B} = a\hat{i} + b\hat{j}$$

$$\vec{B} + \vec{C} = (a + 3)\hat{i} + (b + 4)\hat{j} = |\vec{C}|\hat{j}$$

$$a + 3 = 0$$

$$b + 4 = |\vec{C}| = \sqrt{3^2 + 4^2} = 5$$

$$\vec{B} = -3\hat{i} + 1\hat{j}$$

$$|\vec{B}| = \sqrt{3^2 + 1^2} = \sqrt{10} = 3.2$$

Problem 5

Vector $\vec{A}$, which is directed along an x axis, is to be added to vector $\vec{B}$, which has a magnitude of $6.0m$. The sum is a third vector that is directed along the y axis, with a magnitude that is 3.0 times that of $\vec{A}$. What is that magnitude of $\vec{A}$? 沿著 x 軸定向的向量 $\vec{A}$ 將會加到大小為 $6.0m$ 的向量 $\vec{B}$ 中。總和是沿著 y 軸方向的第三個向量，其大小是 $\vec{A}$ 的 3.0 倍。

(01小題)

$|\vec{A}|$ = ____ m

08: ANS:=1.9

Solution:

$$\vec{A} + \vec{B} = 3A\hat{j}$$

$$\vec{A} = A\hat{i}$$

$$B = 6m$$

$$\vec{B} = -A\hat{i} + 3A\hat{j}$$

$$B = \sqrt{(-A)^2 + (3A)^2} = 6m$$

$$A = \frac{1}{\sqrt{10}}B = 1.9m$$

Problem 6

Use the definition of scalar product, $\vec{a} \cdot \vec{b} = ab\cos\theta$, and the fact that the $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$ to calculate angle between the two vectors given by $\vec{a} = 3.0\vec{i} + 3.0\vec{j} + 3.0\vec{k}$ and $\vec{b} = 2.0\vec{i} + 1.0\vec{j} + 3.0\vec{k}$. 使用標量積的定義 $\vec{a} \cdot \vec{b} = ab\cos\theta$ ，以及 $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$ 計算由 $\vec{a} = 3.0\vec{i} + 3.0\vec{j} + 3.0\vec{k}$ 和 $\vec{b} = 2.0\vec{i} + 1.0\vec{j} + 3.0\vec{k}$ 。

(01小題)

the angle=_____

09: ANS:=22

Solution:

$$\vec{a} \cdot \vec{b} = ab\cos\phi = a_x b_x + a_y b_y + a_z b_z$$

$$a = |\vec{a}| = \sqrt{3^2 + 3^2 + 3^2} = 5.2$$

$$b = |\vec{b}| = \sqrt{2^2 + 1^2 + 3^2} = 3.74$$

$$\cos\phi = \frac{\vec{a} \cdot \vec{b}}{ab} = \frac{(3)(2) + (3)(1) + (3)(3)}{(5.2)(3.74)} = 0.96$$

$$\phi = 22^0$$

Problem 7

In the product $\vec{F} = q\vec{v} \times \vec{B}$, take $q = 2$, $\vec{v} = 2\hat{i} + 4\hat{j} + 6\hat{k}$ and $\vec{F} = 4\hat{i} - 20\hat{j} + 12\hat{k}$. What then is $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ in unit-vector notation if $B_x = B_y$? 在積 $\vec{F} = q\vec{v} \times \vec{B}$ 中，取 $q = 2$ ， $\vec{v} = 2\hat{i} + 4\hat{j} + 6\hat{k}$ 和 $\vec{F} = 4\hat{i} - 20\hat{j} + 12\hat{k}$ 。如果 $B_x = B_y$ ，則單位向量表示法中的 $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ 是什麼？

(03小題)

$$B_x = \underline{\hspace{2cm}}$$

10: ANS:=-3

$$B_y = \underline{\hspace{2cm}}$$

11: ANS:=-3

$$B_z = \underline{\hspace{2cm}}$$

12: ANS:=-4

Solution:

$$4\hat{i} - 20\hat{j} + 12\hat{k} = 2 \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & 6 \\ B_x & B_y & B_z \end{vmatrix}$$

$$4 = 2(4B_z - 6B_y)$$

$$-20 = 2(6B_x - 2B_z)$$

$$12 = 2(2B_y - 4B_x) = 2(-2B_x)$$

$$B_x = B_y = -3$$

$$B_z = \frac{4 + 12 \times (-3)}{8} = -4$$

Problem 8

A vector $\vec{a}$ of magnitude 10 units and another vector $\vec{b}$ of magnitude 6 units differ in directions by 60° . Find (a) the scale product of the two vectors and (b) the magnitude of the vector product. 大小為 10 個單位的向量 $\vec{a}$ 和另一個大小為 6 個單位的向量 $\vec{b}$ 在方向上相差 60° 。求 (a) 兩個向量的尺度積和 (b) 向量積的大小。

(02小題)

(a)

13: ANS:=30

(b)

14: ANS:=52

Solution:

$$(a) \vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\phi = (10)(6)(\cos 60^\circ) = 30$$

$$(b) |\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\phi = (10)(6)(\sin 60^\circ) = 52$$

nprob= 9 9

Problem 9

There are 2 vectors: $\vec{u} = a\hat{i} + b\hat{j}$, $\vec{v} = x\hat{i} + y\hat{j}$ Answer the following questions with functional inputs. 有 2 個向量：

$\vec{u} = a\hat{i} + b\hat{j}$, $\vec{v} = x\hat{i} + y\hat{j}$ 使用功能輸入回答下列問題。(03小題)

(a) The norm of $\vec{u}$, $|\vec{u}| = \underline{\hspace{2cm}}$ [a,b]

15: ANS:=sqrt(a2+b**2).**

(b) The norm of $\vec{v}$, $|\vec{v}| = \underline{\hspace{2cm}}$ [x,y]

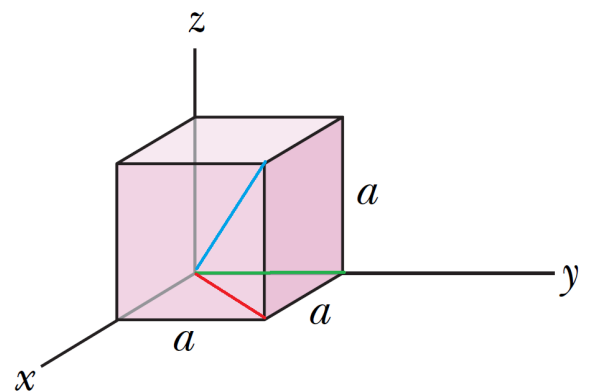
16: ANS:=sqrt(x2+y**2).**

(c) If the angle between $\vec{u}$ and $\vec{v}$ is θ , then $\cos \theta = \underline{\hspace{2cm}}$ [a,b,x,y]

17: ANS:=(a*x+b*y)/(sqrt(a2+b**2)*sqrt(x**2+y**2)).**

Problem 10

In the figure, a cube of edge length a sits with one corner at the origin of an xyz coordinate system. A body diagonal is a line that extends from one corner to another through the center. (a) Determine the angles that the body diagonals make with the adjacent edges (green line in the figure). (b) Determine the angles that the body diagonals make with the adjacent face diagonal (red line in the figure). (c) Determine the length of the body diagonals.



圖中，邊長為 a 的立方體的一個角位於 xyz 座標系的原點。身體對角線是一條通過中心從一個角延伸到另一個角的線。(a) 決定主體對角線與相鄰邊的角度（圖中綠線）。(b) 確定體對角線與相鄰面對角線的夾角（圖中紅線）。(c) 決定身體對角線的長度。(03小題)

(a)the angle=_____ degree

18: **ANS:=54.74**

(b)the angle=_____ degree

19: **ANS:=35.3**

(c)length of the body diagonal=_____ a

20: **ANS:=1.732**