

AM1-L01 學號: _____ 系級: _____ 姓名: _____

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Problem 1

Calculate the following integrations:

1. $I_1 = \int_0^\pi \sin(x) dx =$ _____

2. $I_2 = \int_0^\pi \sin^2(x) dx =$ _____

3. $I_3 = \int_0^\pi x^2 dx =$ _____

$$1 - 2\sin^2 x = \cos 2x$$

$$\sin^2 x = \frac{1}{2} (1 - \cos 2x)$$

(03小題) 01 (1) $I_1 =$ _____ 分數= 10 02 (2) $I_2 =$ _____ 分數= 10 03 (3) $I_3 =$ _____ 分數= 10

1. $\int_0^\pi \sin x dx = \left[-\cos x \right]_0^\pi = -\cos \pi - (-\cos 0) = -(-1) - (-1) = 2$

$$\frac{d(-\cos x)}{dx} = -\left(\frac{d \cos x}{dx}\right) = -(-\sin x) = \sin x$$

2. $\int_0^\pi \sin^2 x dx = \int_0^\pi \frac{1}{2} (1 - \cos 2x) dx = \int_0^\pi \frac{1}{2} dx - \int_0^\pi \frac{1}{2} \cos 2x dx = \frac{\pi}{2} - \left[\frac{1}{4} \sin 2x \right]_0^\pi = \frac{\pi}{2}$
0 為 π , -1 週期的積

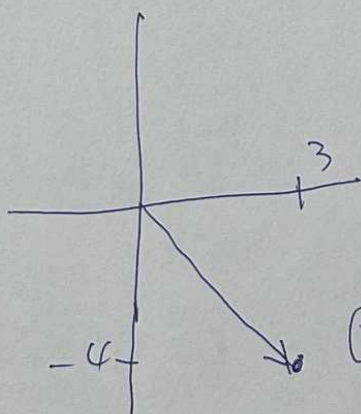
3. $\int_0^\pi x^2 dx = \left[\frac{1}{3} x^3 \right]_0^\pi = \frac{1}{3} \pi^3 - 0 = \frac{1}{3} \pi^3$

Problem 2

The position of a particle is given as

$$\vec{r} = 3\hat{i} - 4\hat{j}$$

What is the position vector in polar coordinate? (02小題) 04 $\rho =$ _____ 分數= 10 05 $\phi =$ _____ 度 分數= 10



$$\rho = \sqrt{x^2 + y^2} = \sqrt{3^2 + (-4)^2} = 5$$

$$\phi = \tan^{-1} \frac{y}{x} = \tan^{-1} \frac{-4}{3} = -\tan^{-1} \frac{4}{3}$$

$$= -53^\circ$$

$$(x, y) = (3, -4)$$

Problem 3

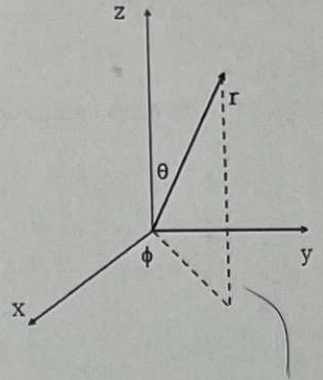
The length of the vector in the figure is 5. $\theta = 45^\circ$, $\phi = 60^\circ$, find the Cartesian coordinates: (03/小題) 06 $x =$ _____ 分數= 10 07 $y =$ _____ 分數= 10 08 $z =$ _____ 分數= 10

$$(r, \theta, \phi) = (5, 45^\circ, 60^\circ)$$

$$x = r \sin \phi \sin \theta = 5 (\sin 60^\circ) \sin 45^\circ \\ = 5 \left(\frac{1}{2}\right) \left(\frac{\sqrt{2}}{2}\right) = \frac{5\sqrt{2}}{4}$$

$$y = r \sin \phi \cos \theta = 5 \sin 60^\circ \cos 45^\circ = 5 \left(\frac{\sqrt{3}}{2}\right) \left(\frac{\sqrt{2}}{2}\right) = \frac{5\sqrt{6}}{4}$$

$$z = r \cos \theta = 5 (\cos 45^\circ) = \frac{5\sqrt{2}}{2}$$

**Problem 4**

Let the position vector (with the tail at the origin) of a moving particle be $\vec{r} = 2t^2\hat{i} - 3t\hat{j} + (t^2 - 2t)\hat{k}$, where t represents time. Answer the following questions. (03/小題)

09 Find the velocity vector $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$ of the particle at time t .

$v_x =$ _____ [t] 分數= 8

10 $v_y =$ _____ [t] 分數= 6

11 $v_z =$ _____ [t] 分數= 6

$$\vec{r} = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$= \frac{d}{dt}(2t^2)\hat{i} + \frac{d}{dt}(-3t)\hat{j} + \frac{d}{dt}(t^2 - 2t)\hat{k}$$

$$= 4t\hat{i} - 3\hat{j} + (2t - 2)\hat{k}$$

$$= v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

$$v_x = 4t, v_y = -3, v_z = 2t - 2$$